Thermal Pressurization of Pore Fluid During Earthquake Slip

Liliana Hromadová

2009

Thermal Pressurization of Pore Fluid During Earthquake Slip

MASTER'S THESIS

Liliana Hromadová

Comenius University

Faculty of Mathematics, Physics and Informatics

Department of Astronomy, Physics of the Earth, and Meteorology

4.1.1 Physics

Supervisor: Prof. RNDr. Peter Moczo, DrSc.

Bratislava 2009

Abstract

Liliana Hromadová

Thermal Pressurization of Pore Fluid During Earthquake Slip

Comenius University
Faculty of Mathematics, Physics and Informatics
Department of Astronomy, Physics of the Earth, and Meteorology

Supervisor: Prof. RNDr. Peter Moczo, DrSc.

Bratislava 2009

93 pages

The presented thesis is intended as a detailed analysis of a candidate dynamic fault weakening mechanism called thermal pressurization of pore fluid, based on the theory of thermoporoelasticity and on the latest geological, laboratory, and theoretical field results.

The thesis starts with a summary of the most important dynamic fault weakening mechanisms, possibly playing a significant role during large tectonic earthquakes, i.e., the thermal pressurization of pore fluid, the flash heating of microscopic asperity contacts, the melting, and the silica gel formation. In the following chapter, an introduction to the theory of thermoporoelasticity is presented, emphasized on a detailed establishment of the theory of linear poroelasticity, and containing a generalization to a thermoporoelastic case. It is the most extensive chapter of the thesis, since the thermal pressurization of pore fluid process can be treated as a thermoporoelastic problem. In the next, final chapter, a modified physical model of the thermal pressurization of pore fluid process is proposed. The chapter starts with a summary of latest geological, laboratory, and theoretical findings regarding the thermal pressurization process, resulting in model properties, constraints and assumptions, and in a set of appropriate values of model parameters. Then the geometry, material properties, physical mechanism, and governing equations of the model are presented. One of the governing equations, which describes the pore fluid pressure variations, differs from the commonly used one - it contains two additional non-linear terms. The performed dimensional analysis suggests

that at least one of the two non-linear terms cannot be omitted in the governing equation.

Abstrakt

Liliana Hromadová

Teplom indukované zvýšenie tlaku tekutiny v póroch

počas sklzu pri tektonickom zemetrasení

Univerzita Komenského v Bratislave
Fakulta matematiky, fyziky a informatiky
Katedra astronómie, fyziky Zeme a meteorológie

Školiteľ: Prof. RNDr. Peter Moczo, DrSc.

Bratislava 2009

93 strán

Predkladaná diplomová práca je zameraná na detailnú analýzu fyzikálneho procesu teplom indukovaného zvýšenia tlaku tekutiny v póroch počas sklzu pri tektonickom zemetrasení. Uvedený proces je dôležitým kandidátom na vysvetlenie dynamického oslabovania tektonického zlomu počas zemetrasenia. Analýza procesu je založená na teórii termoporoelasticity a na najnovších výsledkoch geologického, laboratórneho a teoretického výskumu relevantného pre skúmaný proces.

Práca začína sumarizáciou a stručným popisom najdôležitejších potenciálnych mechanizmov na vysvetlenie dynamického oslabovania zlomu počas zemetrasenia. Sú to nasledovné štyri mechanizmy: teplom indukované zvýšenie tlaku tekutiny v póroch, náhly ohrev a oslabenie mikroskopických kontaktov, natavenie horniny, formovanie kremičitého gélu. Druhá kapitola je koncipovaná ako úvod do teórie termoporoelasticity s dôrazom kladeným na podrobné vybudovanie teórie lineárnej poroelasticity, ktorá je následne zovšeobecnená na termoporoelastický prípad. Ide o najobsiahlejšiu kapitolu práce. Podrobnosť a rozsiahlosť výkladu je motivovaná tým, že skúmaný proces teplom indukovaného zvýšenia tlaku tekutiny v póroch možno klasifikovať ako termoporoelastický problém. V záverečnej kapitole je navrhnutý modifikovaný fyzikálny model skúmaného procesu. Kapitola začína sumarizáciou najnovších geologických, laboratórnych a teoretických výsledkov týkajúcich sa procesu. Následne je navrhnutý

súbor hodnôt modelových parametrov. Kapitola pokračuje prezentovaním geometrie, materiálových vlastností, mechanizmu a riadiacich rovníc modelu. Riadiace rovnice sú odvodené z rovníc termoporoelasticity aplikovaním vlastností fyzikálneho modelu. Riadiace rovnice sú nakoniec transformované do bezrozmerného tvaru a jednotlivé členy sú kvantifikované na základe hodnôt modelových parametrov. Rozmerová analýza ukazuje, že nelineárne členy v rovnici pre variácie teploty možno zanedbať a jej linearizovaný tvar je teda postačujúci na popis skúmaného procesu. Tento výsledok je v zhode s literatúrou. Avšak analogická rozmerová analýza aplikovaná na druhú rovnicu, t.j. rovnicu pre variácie tlaku tekutiny v póroch, ukazuje, že minimálne jeden nelineárny člen v rovnici nemožno zanedbať. Preto sa domnievame, že na korektný popis procesu teplom indukovaného zvýšenia tlaku tekutiny v póroch nemožno použiť linearizovaný tvar rovnice, ktorý je bežne používaný v dostupných fyzikálnych modeloch procesu.

Kľúčové slová: tektonické zemetrasenia, dynamické oslabovanie zlomu, ohrev trením, zvýšenie tlaku tekutiny v póroch, termoporoelasticita.

Foreword

Understanding the mechanisms responsible for the dynamic weakening of tectonic faults during earthquakes is still a great challenge in geophysics, although the earthquake source dynamics, probably pioneered with the spring-and-box model of Burridge and Knopoff (1967), has been studied for more than 40 years. At the present level of knowledge, four different dynamic fault weakening mechanisms may possibly act during large tectonic earthquakes: thermal pressurization of pore fluid, flash heating of microscopic asperity contacts, melting, and silica gel formation. The first two of them seem to be the primary weakening mechanisms, probably acting in combination, as supported by strong experimental and theoretical indications.

Owing to these facts, we decided to focus on the process of thermal pressurization of pore fluid in the presented thesis. The primary future goal is to include the thermal pressurization process in the existing numerical model on rupture propagation, developed by the team of numerical modeling of seismic wave propagation and earthquake motion in the Division of the Physics of the Earth under supervision of Professor Peter Moczo.

The analysis is based both on the theory of thermoporoelasticity and on the published articles on the thermal pressurization process. Mainly the books of Charlez (1991), Detournay and Cheng (1993), Wang, H. (2000), Coussy (2004), and the articles of Biot (1941), Lachenbruch (1980), Palciauskas and Domenico (1982), Mase and Smith (1985), McTigue (1986), Andrews (2002), Bizzarri and Cocco (2006a), Rempel (2006), Rempel and Rice (2006), Rice (2006) are followed in the thesis.

The text is divided into three chapters - starting with an introductory text on the dynamic weakening mechanisms of tectonic earthquakes, followed by a detailed introduction in the theory of thermoporoelasticity, and ending with a physical model of the investigated process. The given text structure was motivated by the fact that a detailed development and presentation of the physical model of the thermal pressurization of pore fluid process starting from the theory of thermoporoelasticity is lacking in the literature.

Our effort results in

- presentation of an introductory text on the theory of thermoporoelasticity,
- concise overview of the latest geological, laboratory, and theoretical findings regarding the thermal pressurization of pore fluid process,
- proposal of a modified physical model of the process,
- derivation of the governing equations of the process.

One of the derived governing equations differs from the commonly used one. Whereas the commonly used equation is linear, we propose a non-linear equation. According to the presented dimensional analysis, at least one non-linear term cannot be omitted in the governing equation. Therefore, we think that the generally used linearized equation is not appropriate for the thermal pressurization of pore fluid process, and the non-linear one proposed here should be used instead.

Contents

X

Notation index

(the SI physical unit is given in square braces, a reference to the definition is given in round braces)

A_p net area of pores per unit bulk volume $[m^2]$

A_0 surface area of the bulk volume $[m^2]$

B Skempton coefficient (2.89)

G shear modulus $[Pa]$

G^* shear modulus of elastic medium $[Pa]$

H inverse of poroelastic expansion coefficient $[Pa]$

J Jacobian of transformation (2.14)

K drained bulk modulus $[Pa]$ (2.51)

$\dot{K}$ rate of kinetic energy $[J\ s^{-1}]$

K_f pore fluid bulk modulus $[Pa]$ (2.71)

K_p drained pore bulk modulus $[Pa]$

K_s solid bulk modulus $[Pa]$ (2.65)

K_s' unjacketed bulk modulus $[Pa]$ (2.59)

K_s'' unjacketed pore modulus $[Pa]$ (2.60)

K_u undrained bulk modulus $[Pa]$

M Biot modulus $[Pa]$ (2.83)

P_c confining pressure $[Pa]$ (2.40)

P_d differential pressure $[Pa]$ (Terzaghi's effective pressure) (2.39)

P_e effective pressure $[Pa]$ (2.42)

P_e' Terzaghi's effective pressure (differential pressure) $[Pa]$ (2.39)

δQ heat supply to the system $[J]$

Q_s heat source $[J\ m^{-3}\ s^{-1}]$

Re Reynolds number (2.130)

S entropy $[J\ K^{-1}]$ (2.34)

S_a uniaxial specific storage coefficient $[Pa^{-1}]$ (2.84)

S_γ unjacketed specific storage coefficient (coefficient of fluid content) $[Pa^{-1}]$ (2.88)

S_ε constrained specific storage coefficient $[Pa^{-1}]$ (2.82)

S_σ unconstrained specific storage coefficient $[Pa^{-1}]$ (2.79)

T temperature $[K]$ (2.34)

ΔU slip $[m]$

$\dot{U}$ rate of internal energy $[J\ s^{-1}]$

V_b bulk volume $[m^3]$

V_f pore fluid volume $[m^3]$

V_p volume of pores $[m^3]$

V_s solid volume $[m^3]$

ΔV_s slip rate $[m\ s^{-1}]$

$\dot{W}_e$ rate of work of external forces $[J\ s^{-1}]$

c Kozeny constant

c_b specific heat capacity of the bulk material $[J\ kg^{-1}\ K^{-1}]$

c_e effective volumetric heat capacity $[J\ m^{-3}\ K^{-1}]$ (2.113)

c_f specific heat capacity of the pore fluid $[J\ kg^{-1}\ K^{-1}]$

c_s specific heat capacity of the solid material $[J\ kg^{-1}\ K^{-1}]$

f coefficient of friction

$\vec{f}$ body force per unit bulk volume $[N\ m^{-3}]$

g_m specific free pore fluid enthalpy $[J\ kg^{-1}]$

h_m specific pore fluid enthalpy $[J\ kg^{-1}]$ (2.167)

k (intrinsic) permeability $[m^2]$ (2.102)

k_f pore fluid thermal conductivity $[J\ s^{-1}\ m^{-1}\ K^{-1}]$

k_s solid thermal conductivity $[J\ s^{-1}\ m^{-1}\ K^{-1}]$

k_{th} effective thermal conductivity $[J\ s^{-1}\ m^{-1}\ K^{-1}]$ (2.115)

l characteristic pore size $[m]$

m_f fluid mass content $[kg\ m^{-3}]$ (2.7)

n Lagrangian porosity (2.8)

$\vec{n}_0$ unit normal vector

p pore fluid pressure $[Pa]$

q Darcy's velocity $[m\ s^{-1}]$ (2.127)

q_{th} heat flux $[J\ s^{-1}\ m^{-2}]$ (2.163)

s_0 entropy per unit bulk volume $[J\ K^{-1}\ m^{-3}]$

s_f^0 entropy of the pore fluid per unit fluid volume $[J\ K^{-1}\ m^{-3}]$

s_m specific pore fluid entropy $[J\ K^{-1}\ kg^{-1}]$

t time $[s]$

t_{tot} total slip duration at a point $[s]$

u matrix displacement $[m]$

u_f pore fluid displacement $[m]$

u_m specific pore fluid internal energy $[J\ kg^{-1}]$

u_v bulk internal energy per unit volume $[J\ m^{-3}]$

v seepage velocity $[m\ s^{-1}]$ (2.128)

v_f pore fluid velocity $[m\ s^{-1}]$

v_s solid velocity $[m\ s^{-1}]$

w_c fault core half-width $[m]$

w_s principal slipping zone half-width $[m]$

$\vec{x}$ position vector $[m]$

α Biot-Willis coefficient (effective stress coefficient) (2.57)

α_b drained bulk thermal expansivity $[K^{-1}]$ (2.103)

α_f pore fluid thermal expansivity $[K^{-1}]$ (2.104)

α_{hy} hydraulic diffusivity $[m^2\ s^{-1}]$ (2.99)

α_m fluid mass content thermal expansivity $[K^{-1}]$ (2.110)

α_p pore volume thermal expansivity $[K^{-1}]$ (2.107)

α_s solid thermal expansivity $[K^{-1}]$ (2.105)

α_{th} thermal diffusivity $[m^2\ s^{-1}]$ (2.114)

α_u undrained bulk thermal expansivity $[K^{-1}]$ (2.111)

β effective stress coefficient for the pore volume (2.58)

β_b drained bulk compressibility $[Pa^{-1}]$ (2.47)

β_b^v uniaxial bulk compressibility $[Pa^{-1}]$ (2.52)

β_{bp} bulk volume expansion coefficient $[Pa^{-1}]$ (2.48)

β_f pore fluid bulk compressibility $[Pa^{-1}]$ (2.70)

β_{pP_c} drained pore compressibility $[Pa^{-1}]$ (2.49)

β_{pp} pore volume expansion coefficient $[Pa^{-1}]$ (2.50)

β_s solid bulk compressibility $[Pa^{-1}]$

δ	increment in the corresponding quantity
δ_{ij}	Dirac delta function
ε_{ij}	strain (2.1)
ε_{kk}	bulk volumetric strain (2.2)
ε_{kk}^f	pore fluid volumetric strain (2.4)
ε_{kk}^s	solid volumetric strain (2.5)
ζ	variation of fluid content (2.6)
η	poroelastic stress coefficient (2.86)
Θ	non-dimensional temperature (3.28)
λ	drained Lame's first parameter $[Pa]$
λ^*	Lame's first elastic parameter $[Pa]$
λ_u	undrained Lame's first parameter $[Pa]$
μ	dynamic fluid viscosity $[Pa\ s]$
ν	drained Poisson's ratio (2.75)
ν_u	undrained Poisson's ratio (2.76)
ξ	non-dimensional position (3.28)
Π	non-dimensional pore fluid pressure (3.28)
$\Pi^{'}$	non-dimensional Terzaghi's effective pressure (3.29)
ρ_b	bulk density $[kg\ m^{-3}]$
ρ_f	pore fluid density $[kg\ m^{-3}]$
ρ_s	solid density $[kg\ m^{-3}]$
σ	mean normal stress $[Pa]$ (2.25)
σ_{ij}	(total) stress $[Pa]$
σ_{ij}^*	(total) stress in elastic medium $[Pa]$ (2.45)
$\tilde{\sigma}_{ij}$	effective stress $[Pa]$ (2.37)
$\tilde{\sigma}_{ij}^{'}$	Terzaghi's effective stress $[Pa]$ (2.38)
σ_{ij}^f	pore fluid (macroscopic) partial stress $[Pa]$ (2.30)
σ_{ij}^s	solid (macroscopic) partial stress $[Pa]$ (2.31)
$\hat{\sigma}_{ij}^f$	pore fluid microscopic stress $[Pa]$ (2.27)
$\hat{\sigma}_{ij}^s$	solid microscopic stress $[Pa]$
σ_{oct}	octahedral stress $[Pa]$ (2.24)

List of Figures

List of Tables

1 Introduction

There are several different types of earthquakes: tectonic, volcanic, collapse, and induced. In this thesis, only the *tectonic earthquakes* are treated. They are the most common earthquakes (they represent about 90% of all earthquakes), and result from regional or global tectonic processes.

Tectonic earthquakes. Tectonic earthquakes are natural phenomena accompanied by a sudden release of mechanical energy accumulated in the Earth's lithosphere. A typical tectonic earthquake does not occur by a creation and propagation of a new rupture in an unfaulted, intact rock. Instead, it occurs predominantly by a frictional sliding along a pre-existing, already weakened zone of finite thickness separating two blocks of the Earth's lithosphere. Such zones are called *fault zones*, or simply, *faults*. They are situated mainly at the boundaries between the Earth's lithospheric plates (the transform plate boundaries, to be specific), but some of the faults are also inside the plates. However, the tectonic earthquake is not a purely frictional phenomenon - the interfacial sliding is accompanied by a destruction of microscopic intermolecular bonds which have been rebuilt during an interseismic period (i.e., the time period between two subsequent earthquakes on the fault). It means that a rupture has to be re-created and propagated on the fault too, although having only a secondary role in the earthquake mechanism (Scholz, 1998). The tectonic earthquake should be therefore understood as a phenomenon comprising both the frictional sliding and the rupture creation and propagation mechanisms. (For brevity, the term earthquake instead of tectonic earthquake is hereafter used.)

Cause and mechanism of earthquakes. During the interseismic period, considerable amounts of stresses and strains are being accumulated on the fault. It is a consequence of global tectonic processes leading to a large-scale relative motion between the adjacent lithospheric plates. At the same time, however, a part of the contact area of the plates can be at rest due to the static friction. The shear stress is being accumulated at a given point of the fault until it reaches the value of the material strength

1

at the point, given by the friction law and called *frictional strength.* "Should the shear stress exceed the frictional strength at the point, a slip, i.e., a relative displacement of the two fault faces occurs" (Moczo et al., 2007). The subsequent evolution of the shear stress follows the friction law. It is a well established fact that the shear stress gradually decreases with the increasing slip at the point during an earthquake. It implies that the decrease has to be due to some dynamic weakening mechanisms. The existence of strong dynamic fault weakening mechanisms is also supported by the latest friction experiments performed under coseismic slip-rates and nearly coseismic confining stresses, which clearly imply that the fault strength under coseismic conditions is much lower than that under conditions of lower slip rates and lower confining stresses.

Dynamic fault weakening mechanisms. Several mechanisms have been proposed so far to explain the dynamic fault weakening behavior during earthquakes. According to the latest theoretical results, and data from laboratory experiments and in-situ geological measurements, the following mechanisms are the most possible candidates for explanation of the dynamic fault weakening behavior (Rice and Cocco, 2006):

- Thermal pressurization of pore fluid.
- Flash heating of microscopic asperity contacts.
- Melting.
- Silica gel formation.

The first three mechanisms listed above are of thermal nature. The latest field observations of mature, highly slipped faults (e.g., the San Andreas fault system in California, or the Median Tectonic Line fault system in Japan) suggest that earthquake slips are accommodated primarily within extremely thin slipping zones ($\approx 1\ mm$). Given to it the fact that slips on mature faults are relatively rapid, with a typical slip-rate of $1\ ms^{-1}$ (Heaton, 1990), a considerable amount of frictional heat is assumed to be generated on the fault during slip. Therefore, the primary weakening mechanisms are likely to be thermal mechanisms induced by *frictional heating* (Fialko, 2004), i.e., heating due to the interfacial sliding on the fault.

Also some other dynamic weakening mechanisms have been proposed, e.g., elastohydrodynamic lubrication (Brodsky and Kanamori, 2001), acoustic fluidization (Melosh, 1996), normal interface vibrations on bimaterial faults (Brune et al., 1993), and normal stress reduction near rupture tip from elastic mismatch (Weertman, 1963), but none of them now seems to be efficient in the dynamic fault weakening process.

Thermal pressurization of pore fluid. This mechanism assumes that fluids (probably water) are present in fault zones, filling the interconnected pores of fault rocks. This assumption might hold, because the majority of earthquakes occur below the water table, and crustal rocks have typically a porous structure. Since the thermal expansion coefficient of water is approximately one hundred times greater than that of the rock, while compressibilities of the water and of the rock are of equal order, the water should expand much more than the porous rock due to the temperature increase caused by frictional heating. However, the expansion of the water in the pores is suppressed by the surrounding solid cage of the rock, therefore the water becomes pressurized. This scenario holds unless the rock is highly permeable, so that the water is being drained away rather than being effectively pressurized, or unless the dilatancy of the rock (inelastic pore volume deformation induced by the rupture propagation and fault slip) overwhelms the thermal expansion effect. If the effective stress law is still valid at high coseismic slip rates, an increase in water pressure would lead to a decrease in the shear strength of the fault, according to the friction law.

The thermal pressurization mechanism was proposed by Sibson (1973), and further elaborated by Lachenbruch (1980), Raleigh and Everden (1981), Mase and Smith (1985), Mase and Smith (1987), Lee and Delaney (1987), Kanamori and Heaton (2000), Andrews (2002), Wibberley (2002), Cocco and Bizzarri (2004), Noda (2004), Andrews (2005), Noda and Shimamoto (2005), Sulem et al. (2005), Bizzarri and Cocco (2006a,b), Rempel and Rice (2006), Rice (2006), Suzuki and Yamashita (2006), Ghabezloo and Sulem (2008a,b), Noda et al. (2008).

Flash heating of microscopic asperity contacts. While at a macroscopic level the sliding surfaces appear to fit tightly together throughout the whole contact area,

from a microscopic viewpoint one would clearly recognize that the sliding surfaces are rough and connected only through several distinct points. These points, called asperity contacts or *asperities*, support much larger stresses than the average macroscopic stress acting on the contact area. Therefore, the local rate of heat production at the asperity contacts is very large during sliding, leading to a sufficiently high local increase in temperature to cause a diminution of contacts strength. Under conditions of sufficiently fast slipping, only a thin zone adjoining the contact is effectively heated, hence the capacity of the net contact area to support normal stresses is almost unchanged. Consequently, the friction coefficient, given as a ratio of the shear stress and the normal stress, is reduced. Recent experimental results suggest that the flash heating becomes a strong weakening mechanism for slip rates above $0.1\ m.s^{-1}$, and the coefficient of friction decreases (approximately as the inverse of the slip-rate) from its low speed values of order 0.6 to final values of order up to 0.2.

Flash heating mechanism was discovered by Bowden and Tabor (1950) who observed flashes of light when looking at a sliding surface of a transparent material. It was further investigated by Archard (1958-1959), Rice (1999), and Beeler and Tullis (2003), Rice (2006), Rempel (2006), and Noda et al. (2008), among others.

Melting. Under assumption of adiabatic shearing, a meter of slip within a few millimeters thick slipping zone should lead to a large increase in temperature (≈ 1000 ℃), thus the temperature became sufficient for melting of the rock (Sibson, 2003). Melting would lead to formation of pseudotachylytes - rocks of basaltic glass appearance formed by frictional melting of the original fault rocks. There is a relatively rare observational evidence for pseudotachylytes, far less than expected. It implies that (at least) one of the following facts is valid:

- Pseudotachylytes are not or rarely preserved in fault zones.
- Slip occurs at much lower stress levels than predicted by Byerlee's law (Noda et al., 2008), thus melting does not occur or is rare, and the pseudotachylytes are not or rarely formed.
- Pseudotachylytes remain unreported primarily due to difficulty in identifying very thin or reworked pseudotachylytes in the fault zones (Kirkpatrick et al., 2009).

- Melting does not occur or has a negligible effect on the fault weakening, since another mechanism(s) reduces the fault strength rapidly once the slip begins.

Because of the lack of strong evidence on the first three points, we accept the last point validity in this thesis.

A further fact that makes the melting phenomenon during earthquakes questionable is that friction experiments performed at coseismic slip rates with materials obtained from active fault zones lead to a dynamic weakening and to a temperature rise, but no melting is observed.

The mechanism of frictional melting was proposed by Jeffreys (1942), and McKenzie and Brune (1972).

Silica gel formation. This mechanism assumes that both water and quartz (silica) are present in fault zones. Then, fine silica particles are assumed to be formed by a granulation process during interfacial sliding, adsorbing water on their surfaces and thus forming a *silica gel*. The silica gel acts as a lubricant, hence it is lowering the shear strength of the fault. If the slipping was stopped, the silica gel would consolidate into a strong, amorphous solid. However, the particle bonding is continuously disrupted during the slipping (thixotropic response). The silica gel is weak, of small viscosity, and deforms at relatively low strength (Rice and Cocco, 2006). Recent experiments have shown that the friction coefficient of rocks rich in quartz can decrease to values as low as 0.1 at slip rates up to $0.1 \; m \; s^{-1}$ over meters of slip, and that it decreases systematically with increasing silica content (Silva et al., 2004). However, the ranges for the slip and slip-rate in which the mechanism is efficient have not yet been determined, and further experiments are needed to identify the possible role of the silica gel formation in the dynamic fault weakening process.

Even if the silica gel formation mechanism occurs during some earthquake events, it could not serve as the only explanation of the dynamic fault weakening process, since there is some evidence on the dynamic weakening observed in friction experiments performed with low silica content rocks.

Silica gel formation mechanism was discovered by Goldsby and Tullis (2002), and further studied by Di Toro et al. (2004).

At the present level of knowledge, the thermal pressurization of pore fluid and the flash heating of microscopic asperity contacts seem to be the two primary fault weakening mechanisms (assumed to act in combination) (Noda et al., 2008), at least during *large earthquakes* (as suggested by Kanamori and Heaton (2000)) occurring on mature faults (i.e., highly slipped faults) and at about typical seismogenic depths ($7\ km$).

Focus of this thesis.

In this thesis, the mechanism called thermal pressurization of pore fluid is investigated, as it is an important candidate for explaining the dynamic fault weakening process during earthquakes. Unless said otherwise, the analysis is constrained to mature *strike-slip faults* (i.e., approximately vertically oriented faults with a predominant horizontally component of interfacial motion) and large *shallow earthquakes* (occurring at depths $\leq 30\ km$; most of them, however, occur at depths $< 20\ km$).

The analysis of the process is based on the theory of thermoporoelasticity, an introduction to which is given in the next chapter, and on the latest results of geological, laboratory and theoretical investigations regarding the thermal pressurization process. As a result, a physical model of the thermal pressurization of pore fluid process with proper governing equations is proposed in the thesis.

2 Introduction to Linear Poroelasticity

with Extension to Thermoporoelasticity

2.1 Overview

Theory of poroelasticity is concerned with *small reversible deformations* of porous solid materials whose elastic behavior is influenced by a fluid filling the pores. In principle, it is an extension of elasticity theory to the situation in which the elastic material is porous and the interconnected pores are saturated with a fluid (the fluid is compressible and viscous in general). The poroelasticity theory deals with a time-dependent coupling between the deformation of the porous solid material, and the fluid pressure variations and the fluid flow within the material.

The assumption of reversibility is crucial because it makes it possible to build up the theory by using the classical thermodynamics. Within that framework, we are dealing with a conservative physical system (all the dissipative forces are neglected), which is in equilibrium when at rest. Any deformation is a departure from the equilibrium state of minimum potential energy. The work done to bring the material from an initial (non-deformed) to a final (deformed) state is independent of the way by which the final state is reached, therefore it can be expressed as a definite function of the adopted variables (e.g., six strain components and the variation of fluid content).

2.1.1 Theory of Linear Poroelasticity

Theory of linear poroelasticity is the poroelasticity theory constrained to porous materials exhibiting a materially linear behavior in the elastic domain. Hence, the constitutive stress - strain relations are linear.

A founding paper on linear poroelasticity, although not the first in the field, was written more than half a century ago by Maurice A. Biot (1941). The very first theory accounting for the influence of the pore fluid on the soil deformation was developed by Karl Terzaghi (1923), based on his laboratory experiments on soil consolidation (the process of a gradual settlement of the soil under an applied load). Terzaghi's one-dimensional theory was generalized to three-dimensions by Biot, who also showed that

Terzaghi's one-dimensional treatment is a special case of his general three-dimensional theory. Later on, Biot introduced the non-linear poroelasticity (Biot, 1973).

Terminology and Notations. In what follows, the term *bulk* (denoted either by a subscript b or by no subscript) is associated with the lump fluid-filled porous material (i.e., the rock). It consists of a porous *matrix* (m) and of a fluid in the pores of the matrix. The fluid in the pores will be called *pore fluid* (f). The matrix is composed of a *solid material* (s). The terms *fluid phase* and *solid phase* are sometimes used instead of the pore fluid and the solid material, respectively. A zero superscript or a zero subscript denotes an initial state. Italics is used, when an important phrase occurs for the first time.
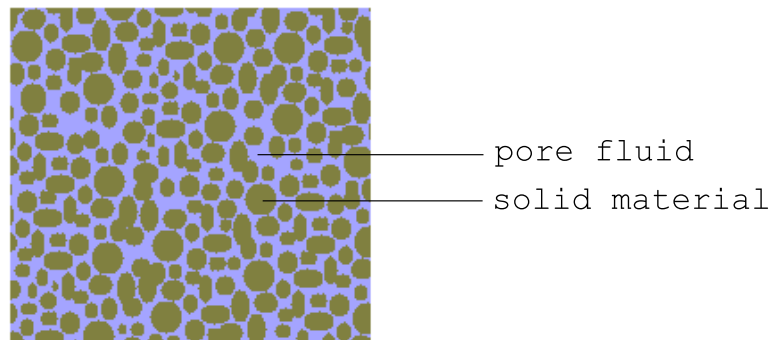
Figure 1. An illustration of the bulk material.

Assumptions. According to the extensively validated, and today widely accepted Biot's works on elasticity and consolidation of porous materials (Biot, 1941, 1955, 1956, 1962), the following basic properties of the rock are assumed (additionally to the essential, already introduced assumptions of small strains, and reversible and linear stress - strain relations):

- The solid material is homogeneous and isotropic (but the bulk material may be not).
- All the pores are interconnected.

In addition to the basic assumptions, the following assumptions are frequently employed in order to simplify the problem:

- Thermoelastic effects are not considered, i.e., there is no functional dependency between the temperature and the stress or strain fields (*isothermal linear poroelasticity*).

- The bulk material is macroscopically homogeneous and isotropic.

- The interconnected pores are fully saturated with the pore fluid.

- The pores are filled with a single pore fluid (e.g., water) in a single phase (liquid, gaseous or supercritical).

- Chemical effects are not considered.

In this thesis, the last four assumptions are applied.

The homogeneity of the bulk material means that values of material parameters do not vary in space, i.e., they are constant. According to Charlez (1991), the isotropy of the material can be defined by postulating that normal stresses generate only normal strains (i.e., normal stresses are responsible only for volume changes), and shear stresses generate only shear strains (i.e., shear stresses are responsible only for shape changes).

In agreement with the majority of literature on poroelasticity, a macroscopic treatment of the poroelastic behavior is used. The concept of continuum mechanics is adopted, in which a representative cubic element of the linear poroelastic material, called *representative elementary volume* (REV), is introduced. The REV is taken to be large enough compared to the size of the pores (in order to be macroscopically homogeneous), but at the same time small enough compared to the characteristic scale of the investigated macroscopic processes.

2.1.2 Theory of Linear Thermoporoelasticity

Theory of linear thermoporoelasticity, also referred to as *theory of non-isothermal linear poroelasticity*, can be built up as an extension of the (isothermal) linear poroelasticity theory to the case when thermal expansion effects, both of the matrix and of the pore fluid, must be taken into account. The relevant situations are associated either with the temperature changes which are sufficiently high to considerably affect the elastic properties (i.e., the stress and strain fields) of the saturated porous material, or contrariwise, with the deformations affecting the temperature significantly (however, the latter effect is in most cases neglectable in comparison to the first one). A thermoporoe-

lastic material is thus concerned both with thermal and mechanical couplings between the pore fluid and the solid material (the isothermal poroelasticity involves only the mechanical coupling). Within the linear framework, the extension of the poroelasticity theory to the thermoporoelasticity theory is straightforward - it consists in adding a single term accounting for temperature changes into the constitutive equations. Furthermore, the energy conservation laws must be taken into account in order to derive governing equations of thermoporoelasticity.

The classical theory of thermodynamics postulates that the state of any physical system is be determined by a certain number of functions, called *state functions*, which are dependent on a certain number of variables. Under conditions of reversibility, only a single state function is needed to determine the behavior of a given material (i.e., its constitutive equations). The state function is called *thermodynamic potential*. It has a quadratic form if the material is linearly elastic. All the phenomena of heat dissipation (e.g., viscous dissipation within the pore fluid or at the fluid-solid boundary, plastic deformations, damage or rupturing of the material) are omitted, and the only variables that must be considered are the so-called *state variables* (in the irreversible case, also the *internal variables* must be taken into account) (Charlez, 1991).

The theory of linear thermoporoelasticity was introduced by Palciauskas and Domenico (1982) and McTigue (1986), and further developed by Charlez (1991) and Coussy (2004), among others. In the latter book, also the non-linear thermoporoelasticity is treated.

2.2 Variables

Four basic variables, introduced by Biot, are used in the constitutive equations of linear poroelasticity. They can be grouped into two conjugate pairs:

1. *Stress* and *strain*.
2. *Pore fluid pressure* and *variation of fluid content*.

The purpose of such classification of variables is to distinct the dependent and independent variables in the constitutive equations. A pair of the dependent variables is

composed of one variable from each conjugate pair. A pair of the independent variables is composed of the remaining two variables.

Another classification possibility is to distinguish *kinematic variables*,

- strain,
- variation of fluid content,

and *dynamic variables*,

- stress,
- pore fluid pressure.

2.2.1 Strain

Strain tensor for a fluid-saturated linear poroelastic material, under the assumption of small strains, is defined in the same way as for an elastic material,

$$\varepsilon_{ij} \; = \; \frac{1}{2} \left(\frac{\partial \, u_i}{\partial \, x_j} \; + \; \frac{\partial \, u_j}{\partial \, x_i} \right) \; ; \qquad i \, , j \; \in \; \{1, \, 2, \, 3 \} \, , \tag{2.1}$$

where u_i is the i-th component of the *displacement vector*, describing the movement of the porous material with respect to the initial configuration. The definition suggests that the strain tensor is symmetric, i.e., $\varepsilon_{ij} \; = \; \varepsilon_{ji}$. *Shear strains* and *longitudinal strains* are the strain tensor components for which $i \; \neq \; j$ and $i \; = \; j$ in eq. (2.1), respectively. Sometimes it is reasonable to use the term *bulk strain* instead of the term strain, in order to emphasize that it measures the deformation of the bulk material, not the deformation of the solid or of the fluid phase. The *bulk volumetric strain* is defined as the sum of three longitudinal strain components:

$$\varepsilon \; \equiv \; \varepsilon_{kk} \; = \; \frac{\delta \, V_b}{V_b^0} \; = \; \varepsilon_{xx} \; + \; \varepsilon_{yy} \; + \; \varepsilon_{zz} \, . \tag{2.2}$$

Here, V_b and V_b^0 are the bulk volumes in the actual and in the initial state, respectively. Hereafter, δ will denote the increment (i.e., a small positive change) in the corresponding quantity, and the summation convention will be used whenever the repeated indices occur. The bulk volume increment is given as

$$\delta \, V_b \; = \; V_b \; - \; V_b^0 \, . \tag{2.3}$$

Analogously, we define the *pore fluid volumetric strain* (ε_{kk}^f), and the *solid volumetric strain* (ε_{kk}^s) respectively as

$$\varepsilon_{kk}^f \;=\; \frac{\delta\,V_f}{V_f^0}\,, \tag{2.4}$$

$$\varepsilon_{kk}^s \;=\; \frac{\delta\,V_s}{V_s^0}\,, \tag{2.5}$$

where $V_{(f/s)}$ and $V_{(f/s)}^0$ are the net volumes of the pore fluid/solid material within the bulk volume in the actual and in the initial state, respectively.

We adopt the usual sign convention used in the poroelasticity literature, so that extensional strains are taken as positive and compressional as negative (the same convention holds for stresses).

2.2.2 Variation of Fluid Content

This kinematic variable was introduced by Biot (1941) under the term *variation in water content*. The variation of fluid content is defined as the change in the pore fluid volume per unit bulk volume in the non-deformed state, which can be due to a volumetric deformation of the bulk material - leading to a change in the volume of the pores $(\delta\,V_p)$ within the non-deformed bulk volume (V_b^0) and thus to a gain or loss of some fluid from the bulk volume, or due to a change in the pore fluid pressure - leading to a compression or expansion of the pore fluid and thus to a change in the pore fluid volume $(\delta\,V_f)$ within the non-deformed bulk volume. The variation of fluid content is given as follows:

$$\zeta \;=\; \frac{\delta\,V_p \;-\; \delta\,V_f}{V_b^0}\,. \tag{2.6}$$

The pore fluid exchange $(\zeta \neq 0)$ between the reference bulk volume (V_b^0) and the surrounding medium can thus result both from the bulk volume deformation $(\delta\,V_p \neq 0)$, and from the pore fluid pressure change $(\delta\,V_f \neq 0)$.

A positive variation of fluid content $(\zeta > 0)$ corresponds to a gain of the pore fluid in the bulk volume.

Fluid Mass Content. Seeing that the variation of fluid content (ζ) is not a state variable (cannot be determined at a given time, it is defined only for a given interval of time), another quantity which is already a state variable, called *fluid mass content*,

can be used in equations instead. It was defined by Rice and Cleary (1976) as the net mass of the pore fluid per unit bulk volume in the initial state:

$$m_f \quad = \quad \frac{\rho_f \, V_f}{V_b^0} \quad = \quad \rho_f \, n \, . \tag{2.7}$$

Here, ρ_f is the *pore fluid density*, and n denotes the property of the porous material called *porosity*. It is defined as a ratio of the net *volume of the pores* (V_p) within the bulk volume (V_b) and the bulk volume in the initial state (V_b^0):

$$n \quad = \quad \frac{V_p}{V_b^0} \, . \tag{2.8}$$

Sometimes the term *effective porosity* instead of porosity is used in order to emphasize that the porosity accounts only for the interconnected pore space (the possible non-connected pore space is considered as a part of the solid material). Porosity defined by eq. (2.8) is also called *Lagrangian porosity* or *apparent fluid volume fraction*. There is also a different definition of porosity, in which the bulk volume in the actual deformed state (V_b) instead of the bulk volume in the initial state (V_b^0) is used. Then, the porosity is called *Eulerian porosity* and is denoted by ϕ,

$$\phi \quad = \quad \frac{V_p}{V_b} \, . \tag{2.9}$$

Assuming a full fluid saturation of the porous material, an equality $V_p = V_f$ holds, and the Lagrangian and Eulerian porosities can be alternatively defined respectively as

$$n \quad = \quad \frac{V_f}{V_b^0} \, , \tag{2.10}$$

$$\phi \quad = \quad \frac{V_f}{V_b} \, . \tag{2.11}$$

From eqs. (2.10) and (2.11), the following relation between n and ϕ can be derived:

$$n \quad = \quad \phi \, \frac{V_b}{V_b^0} \, . \tag{2.12}$$

Clearly, the Lagrangian porosity (n) and the Eulerian porosity (ϕ) are equal in the initial state $(V_b = V_b^0)$,

$$n_0 \quad = \quad \phi_0 \, . \tag{2.13}$$

The ratio V_b/V_b^0 in eq. (2.12) has a special meaning under conditions of small deformations. It is the *Jacobian of transformation* of the bulk volume from the initial (non-deformed) to the final (deformed) state. It is also called *Jacobian of deformation*.

13

We will use it later to make transformations between Eulerian and Lagrangian reference frames. For this purpose, the denotation is made here:

$$J = \frac{V_b}{V_b^0} . \tag{2.14}$$

Or, following the definition of the bulk volumetric strain (ε_{kk}), given by eq. (2.2), and using eq. (2.3), we can write

$$J = 1 + \varepsilon_{kk} . \tag{2.15}$$

According to eqs. (2.14) and (2.15), the Lagrangian porosity (n) and the Eulerian porosity (ϕ) are related through relation

$$n = \phi J = \phi (1 + \varepsilon_{kk}) . \tag{2.16}$$

Now, let us return to the discussion on the variation of fluid content (ζ), in order to introduce some alternative expressions for it. According to Wang, H. (2000), the following relation holds between the variation of fluid content (ζ) and the fluid mass content (m_f):

$$\zeta = \frac{\delta m_f}{\rho_f^0} . \tag{2.17}$$

Here, ρ_f^0 is the pore fluid density in the initial state, and δm_f denotes the increment of the fluid mass content, given as

$$\delta m_f = m_f - m_f^0 . \tag{2.18}$$

The variation of fluid content (ζ) can be further expressed in terms of the volumetric strain of the bulk material (ε_{kk}) and that of the pore fluid (ε_{kk}^f). For this purpose, the definitions of ε_{kk}, and ε_{kk}^f, given by eqs. (2.2) and (2.4), respectively, are used in the definition of ζ, given by eq. (2.6). Under conditions of *ideal porous material*, introduced by Detournay and Cheng (1993), one obtains an alternative expression for the variation of fluid content:

$$\zeta = n_0 \left(\varepsilon_{kk} - \varepsilon_{kk}^f \right) . \tag{2.19}$$

In order to derive eq. (2.19), we used the relation

$$\frac{\delta V_p}{V_p^0} = \frac{\delta V_b}{V_b^0} , \tag{2.20}$$

which holds for an ideal porous material, i.e., the porous material with a fully interconnected pore space, having the matrix comprised of a homogeneous and isotropic

solid material. The Eulerian porosity does not change ($\delta \phi = 0$) if the ideal porous material is deformed under conditions of constant *differential pressure* (2.39). Then the bulk volumetric strain (ε_{kk}) equals the volumetric strain of the solid material (ε_{kk}^s), and according to eq. (2.20) we obtain,

$$\frac{\delta V_s}{V_s^0} = \frac{\delta V_b}{V_b^0} , \tag{2.21}$$

where the basic relation between the bulk volume (V_b), the pore volume (V_p), and the solid volume (V_s) was used:

$$V_b = V_p + V_s . \tag{2.22}$$

Hence, also the following expression for the variation of fluid content is valid in this special case (see also eq. (2.64) and the comment below it about the ideal porous material):

$$\zeta = n_0 \left(\varepsilon_{kk}^s - \varepsilon_{kk}^f \right) . \tag{2.23}$$

2.2.3 Pore Fluid Pressure

The pore fluid pressure, also called *pore pressure* or *interstitial pressure*, is the pressure of the fluid which fills the interconnected pores of the matrix. Commonly, it is denoted by p. Its most fundamental definition reads (Rice and Cleary, 1976): the pore fluid pressure is the equilibrium pressure that must be exerted on the homogeneous reservoir of the pore fluid which is in contact with a control volume of the bulk material, in order to avoid any pore fluid exchange between the reservoir and the control volume. It is a scalar quantity, since producing an equal force per unit area in all directions.

The definition of the pore fluid pressure (p) restricts the subject of the poroelasticity theory to *quasi-static deformation* processes, in the sense that changes occur at sufficiently slow rate so that a local pressure equilibrium in the REV can be attained.

We use the usual sign convention, in which p greater than atmospheric is positive (i.e., the compressional pressure is taken as positive, which is an opposite convention to that adopted for strains and stresses).

Drained and Undrained Conditions. The presence of a fluid in the interconnected pores of the porous material implies that two limiting conditions associated with the bulk volume deformation can be considered.

Drained conditions correspond to a deformation occurring at the sufficiently slow rate, so that the pore fluid is allowed to flow into or out of the bulk volume without any change in the pore fluid pressure, i.e., $\delta p = 0$.

Undrained conditions correspond to a deformation occurring at the time scale that is too short to allow the pore fluid flow into or out of the bulk volume, thus the variation of fluid content is zero, i.e., $\zeta = 0$ (or equivalently, the change in the fluid mass content is zero, $\delta m_f = 0$).

2.2.4 Stress

The stress expresses the force acting per unit area of the bulk material, similarly as in the classical theory of linear elasticity. The stress tensor component σ_{ij} is defined as the force in the j-th direction, with which the material with greater x_i acts through a unit area of the bulk material with normal in the i-th direction on the material with lesser x_i. Since it describes the force acting per unit area of the bulk material and not of the solid or fluid phase alone, it is also called *total stress*. The stress tensor is symmetric, $\sigma_{ij} = \sigma_{ji}$, hence the state of stress in the given point of the poroelastic material is fully described by six stress components. *Shear stresses* and *normal stresses* are the stress tensor components for which $i \neq j$ and $i = j$, respectively. The *octahedral stress* is defined as the sum of three normal stresses,

$$\sigma_{oct} = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) , \tag{2.24}$$

and the *mean normal stress* is given as the octahedral stress divided by three:

$$\sigma = \frac{1}{3} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) . \tag{2.25}$$

The total stress (σ_{ij}) is sometimes called *confining stress*, because the only way in which the total stress (σ_{ij}) can be applied to the bulk material is an outwardly applied surface force. Unlike it, the pore fluid pressure (p) acts on the individual solid grains comprising the porous matrix inside the bulk material.

Compressional stresses are negative in the sign convention used in this thesis.

Concept of Partial Stresses. In the theory of poroelasticity, the total stress (σ_{ij}) can be separated into two parts, following the concept of partial stresses introduced by Biot (1955). Let us constrain to a representative elementary volume of the bulk material

(REV). While from the macroscopic viewpoint a single value of the total stress (σ_{ij}) can be assigned to the REV, at the microscopic level the fluid and the solid phases are distinguishable and an individual stress value can be assigned to each phase. Let us denote the average microscopic stress acting in the solid phase by $\hat{\sigma}_{ij}^s$, and the average microscopic stress acting in the fluid phase by $\hat{\sigma}_{ij}^f$. Hereafter, we will refer to them as the *solid microscopic stress* and the *pore fluid microscopic stress*, respectively. Then the macroscopic total stress (σ_{ij}) can be expressed in terms of the microscopic stresses as follows:

$$\sigma_{ij} \;=\; \phi\,\hat{\sigma}_{ij}^f \;+\; (\,1\,-\,\phi\,)\,\hat{\sigma}_{ij}^s\,. \tag{2.26}$$

The pore fluid microscopic stress ($\hat{\sigma}_{ij}^f$) can be further expressed as

$$\hat{\sigma}_{ij}^f \;=\; \tau_{ij}^f \;-\; p\,\delta_{ij}\,, \tag{2.27}$$

where δ_{ij} denotes *Dirac delta function*, and τ_{ij}^f denotes the *pore fluid viscous shear stress*, including both the internal viscous stress related to the internal pore fluid friction and the viscous stress at the fluid-solid interface. Generally, the viscous stress within the pore fluid is much smaller than that at the interface. Therefore, τ_{ij}^f will denote only the *pore fluid viscous shear stress at the fluid-solid interface* hereafter, and the pore fluid viscosity will enter the problem only through the drag force at the solid-fluid interface under conditions of non-zero fluid flow. The additional assumption we use is that normal viscous stresses are omitted, since only shear flows (no extensional or compressional flows) will be considered later on. Since the fluids can support shear stresses only when a relative motion within them exists continuously, i.e., under conditions of non-zero pore fluid flow, in the state of static equilibrium the viscous shear stress becomes zero ($\tau_{ij}^f \,=\, 0$) and the pore fluid microscopic stress ($\hat{\sigma}_{ij}^f$) is then represented only by the isotropic pore fluid pressure (p):

$$\hat{\sigma}_{ij}^f \;=\; -\,p\,\delta_{ij}\,, \tag{2.28}$$

Hence, the expression for the macroscopic total stress (2.26) under equilibrium conditions is modified as follows:

$$\sigma_{ij} \;=\; -\,\phi\,p\,\delta_{ij} \;+\; (\,1\,-\,\phi\,)\,\hat{\sigma}_{ij}^s\,. \tag{2.29}$$

It should be noted that the above equation is approximately valid also in the case of non-zero fluid flow, because the viscous shear stress at the fluid-solid interface (τ_{ij}^f)

17

is generally negligible in comparison with the hydrostatic pore fluid pressure (p) (Papanastasiou et al., 2000).

Let us make the following denotations, in order to pass from the microscopic to the macroscopic point of view:

$$\sigma_{ij}^f = -\phi\, p \,. \tag{2.30}$$

$$\sigma_{ij}^s = (1 - \phi)\, \hat{\sigma}_{ij}^s \,. \tag{2.31}$$

Then, eq. (2.29) can be rewritten as

$$\sigma_{ij} = \sigma_{ij}^f\, \delta_{ij} + \sigma_{ij}^s \,, \tag{2.32}$$

which is the commonly used expression for the total stress (σ_{ij}), where σ_{ij}^s is called *solid partial stress* and σ_{ij}^f is called *pore fluid partial stress*. When the total stress acting on the REV of the bulk material is σ_{ij}, the solid partial stress (σ_{ij}^s) quantify that part of σ_{ij} which is supported by the solid phase. The fluid partial stress (σ_{ij}^f) has an analogous meaning. The introduction of partial stresses is, however, not necessary in order to build up the theory of poroelasticity.

Effective Stress. Furthermore, the stress called *effective stress* is used in the theory of poroelasticity. The effective stress, hereafter denoted by $\tilde{\sigma}_{ij}$, represents that part of the total stress σ_{ij}, which produces the strain in the matrix, i.e., the part contributing to the deformation of the bulk material. The pore fluid pressure (p) acts in a counteractive manner to the total stress (σ_{ij}) during the bulk material deformation, hence only a part of the total stress effectively participate in the deformation of the bulk material. That part is called effective stress $(\tilde{\sigma}_{ij})$. Terzaghi (1923) was the first who realized this from his experiments on soil consolidation (i.e., the process of gradual settlement of soil under applied load). He also theoretically introduced the effective stress in the one-dimensional form. Therefore, the effective normal stress is often referred to as *Terzaghi's effective pressure*. A more detailed treatise on the effective stress principle is given later on. It is of great importance in the theory of poroelasticity.

2.2.5 Variables of Linear Thermoporoelasticity

As already introduced, the linear thermoporoelasticity involves only one state function called thermodynamic potential, which depends on a certain number of state variables.

The laws of thermodynamics imply that only three observable variables (i.e., the experimentally measurable ones) exist under given conditions: the strain (ε_{ij}), the fluid mass content (m_f), and the *temperature*. Hence, in addition to the four variables associated with a problem of isothermal linear poroelasticity, the temperature is chosen as the fifth variable required to introduce the constitutive equations of linear thermoporoelasticity.

Temperature. The definition of temperature (also called *absolute temperature*) is based on the theory of thermodynamics and requires a relatively complex treatment. Therefore, the temperature will be introduced simply through the second law of thermodynamics in what follows. This approach is sufficient for our purposes.

The second law of thermodynamics can be stated in various equivalent forms. Its most fundamental mathematical formulation for a general physical system, when both reversible and irreversible processes are taken into account, reads:

$$d\,S \;\; \geq \;\; \frac{\delta\,Q}{T}\,. \tag{2.33}$$

Here, T is the temperature of the system, $\delta\,Q$ is the external supply of *heat* into the system, S denotes the state variable called *entropy*, and $d\,S$ denotes its differential. The entropy of the system (S) is, broadly speaking, a measure of the system's disorder (or, more precisely, of the disorder of the molecules comprising the system). Eq. (2.33) states that the entropy of a closed system never decreases.

In the case of purely reversible processes, the inequality in eq. (2.33) is transformed into equality. Thus, the following relation is valid in the theory of thermoporoelasticity:

$$d\,S \;\; = \;\; \frac{\delta\,Q}{T}\,. \tag{2.34}$$

2.3 Constitutive Equations

As already mentioned, the key concept of poroelasticity is the coupled mechanical behavior between the solid and the fluid phases. Two basic coupling phenomena may occur (Wang, H., 2000):

1. Matrix-to-fluid coupling, characterized by the change in the pore fluid pressure (δp) or in the fluid mass content (δm_f) due to the change in total stress ($\delta \sigma_{ij}$).

2. Fluid-to-matrix coupling, characterized by the change in the bulk volume (δV_b) due to the change in the pore fluid pressure (δp) or in the fluid mass content (δm_f).

Within the linear framework, the coupling is mathematically expressed through *constitutive equations of linear poroelasticity*, introduced by Biot (1941). They generally consist of seven linear equations and contain four independent material parameters. The number of constitutive equations is reduced to four in the principal coordinate system, where the shear stresses and the shear strains are zero. Further reduction to only 2 linear equations containing only 3 independent material coefficients is achieved in the case of isotropic stress and strain fields.

As introduced in the previous section, four variables appear in the constitutive equations. They comprise the set of seven linear equations in the following way. Six equations express the tensor components of the first dependent variable, which can be either the stress (σ_{ij}) or the strain (ε_{ij}), as a linear combination of two independent variables. The seventh equation expresses the second dependent variable, which can be either the pore fluid pressure (p) or the variation of fluid content (ζ), as a linear combination of two independent variables. The independent variables are the two variables which remain of the full set of four variables after choosing the two dependent variables. As already discussed in the section on variables, there are two conjugate pairs of variables from which the two dependent variables are chosen, each dependent variable from another conjugate pair. Since the two conjugate pairs of variables are each composed of two variables, four permutations of dependent and independent variables exist. Consequently, there are four different formulations of the constitutive equations of linear poroelasticity:

- *Pure stiffness formulation* - the stress (σ_{ij}) and the pore fluid pressure (p) are the dependent variables, and the strain (ε_{ij}) and the variation of fluid content (ζ) are the independent variables.

- *Mixed stiffness formulation* - the stress (σ_{ij}) and the variation of fluid content (ζ) are the dependent variables, and the strain (ε_{ij}) and the pore fluid pressure (p) are the independent variables.

- *Pure compliance formulation* - the strain (ε_{ij}) and the variation of fluid content (ζ) are the dependent variables, and the stress (σ_{ij}) and the pore fluid pressure (p) are the independent variables.

- *Mixed compliance formulation* - the strain (ε_{ij}) and the pore fluid pressure (p) are the dependent variables, and the stress (σ_{ij}) and the variation of fluid content (ζ) are the independent variables.

The term pure represents the fact that the two dependent variables have same physical units.

In what follows, Biot's constitutive equations of linear poroelasticity are given in their general formulation, i.e., consisting of seven scalar equations and containing four independent material parameters. The mixed stiffness formulation is used. The other three formulation can be obtained from it after some simple algebraic manipulations.

An important note should be made. Equations of poroelasticity are mostly written in terms of absolute quantities (e.g., p). Sometimes, increments of the quantities appear in the equations instead (e.g., $\delta\,p$; $\delta\,p = p - p_0$). However, those two expressions are equivalent ones (i.e., $p \equiv \delta\,p$), and the absolute quantities should be understood as representing the incremental values measured with respect to the initial state ($p \equiv p - p_0$).

The constitutive equations for an isotropic, fluid-saturated, linear poroelastic material under isothermal conditions are given as

$$
\begin{aligned}
\sigma_{ij} &= 2\,G\,\varepsilon_{ij} + \lambda\,\varepsilon_{kk}\,\delta_{ij} - \alpha\,p\,\delta_{ij}\,, \\
\zeta &= \alpha\,\varepsilon_{kk} + \frac{1}{M}\,p\,,
\end{aligned}
\tag{2.35}
$$

where G, λ, α, and M are four independent poroelastic parameters (will be defined in the next section); $i, j, k \in (1, 2, 3)$. Alternatively, eq. (2.35) can be written explicitly for the six stress components, thus we obtain a full set of seven constitutive equations of linear poroelasticity:

21

$$\sigma_{xx} \;=\; 2\,G\,\varepsilon_{xx} \;+\; \lambda\,\varepsilon_{kk} \;-\; \alpha\,p\,,$$

$$\sigma_{yy} \;=\; 2\,G\,\varepsilon_{yy} \;+\; \lambda\,\varepsilon_{kk} \;-\; \alpha\,p\,,$$

$$\sigma_{zz} \;=\; 2\,G\,\varepsilon_{zz} \;+\; \lambda\,\varepsilon_{kk} \;-\; \alpha\,p\,,$$

$$\sigma_{xy} \;=\; 2\,G\,\varepsilon_{xy}\,, \tag{2.36}$$

$$\sigma_{xz} \;=\; 2\,G\,\varepsilon_{xz}\,,$$

$$\sigma_{yz} \;=\; 2\,G\,\varepsilon_{yz}\,,$$

$$\zeta \;=\; \alpha\,\varepsilon_{kk} \;+\; \frac{1}{M}\,p\,.$$

The first six equations, or equivalently, their concise representation given by the first equation of (2.35), are sometimes referred to as *Hooke's law for poroelastic medium.*

The assumption of bulk material isotropy is not essential and the above equations can be easily extended to the anisotropic case, which might be of practical importance since many geologic materials in Earth's crust are anisotropic. However, also in the simplest case of anisotropy, which is the transverse isotropy (the material is axially symmetric about one axis), the total number of independent parameters is increased (from four) to eight, thus the problem becomes much more complicated.

2.3.1 Effective Stress Concept

As introduced in the previous section, the stress called effective stress ($\tilde{\sigma}_{ij}$) is of great importance in the theory of poroelasticity. The effective stress is defined as the only stress governing the strain of the matrix. In other words, all the measurable deformation of the bulk material is exclusively due to the effective stress. Mathematically, it is defined as the linear combination of the total stress (σ_{ij}) and the pore fluid pressure (p):

$$\tilde{\sigma}_{ij} \;=\; \sigma_{ij} \;+\; \alpha\,p\,\delta_{ij}\,. \tag{2.37}$$

Here, α denotes the poroelastic parameter called *Biot-Willis coefficient* (called also *Biot coefficient* or *effective stress coefficient*). Its value is determined experimentally for the particular rock type, mostly ranging between n and 1 (Berryman, 1992). (See the next section for the definition and details on α.) Note that the pore fluid pressure term in the definition of the effective stress (2.37) is added only to the normal stresses,

since the pore fluid pressure cannot produce any shearing strain because of the assumed isotropy of the bulk material.

It was Terzaghi (1923) who introduced the effective stress concept from his experiments. The definition of *Terzaghi's effective stress* can be obtained from the general definition (2.37) by setting $\alpha = 1$,

$$\tilde{\sigma}'_{ij} = \sigma_{ij} + p\,\delta_{ij}\,, \tag{2.38}$$

which corresponds to the special conditions under which both the solid and the pore fluid phases of the rock are incompressible (Nuth and Laloui, 2008). In fact, Terzaghi's original definition was even more simplified, as it was given in principal coordinates (where only the normal stress components are non-zero), under the assumption of an isotropic stress field ($\sigma_{xx} = \sigma_{yy} = \sigma_{zz}$). In this case, the effective stress is called *Terzaghi's effective pressure* and is given as

$$P'_e = P_c - p\,, \tag{2.39}$$

where P_c denotes the so-called *confining pressure*, defined as

$$P_c = -\sigma\,, \tag{2.40}$$

which leads to the alternative expression for (2.39):

$$P'_e = -\sigma - p\,. \tag{2.41}$$

Note that the negative sign in the last two equations ($-\sigma$) is due to the sign convention, in which the tensile stresses and the compressional pressures are positive. Terzaghi's effective pressure (P'_e) is also called *differential pressure*, denoted by P_d ($P'_e \equiv P_d$).

In general, the Biot-Willis coefficient is not equal to one ($\alpha \neq 1$), and the *effective pressure* is defined as

$$P_e = P_c - \alpha\,p\,, \tag{2.42}$$

which reduces to Therzaghi's effective pressure (P'_e) if $\alpha = 1$.

According to most experiments, the simplified relation (2.38), or (2.39) in the isotropic stress case, seems to be approximately correct for the majority of rocks.

The concept of effective stress is naturally contained in Biot's theory. Thus in principle we do not need Terzaghi's definition of the effective stress, since the general definition given by eq. (2.37) follows quite straightforwards from Biot's equations. Rewriting

23

the constitutive equations for the stress components, given by the first equation of (2.35), such that only the strain terms remain on the r.h.s., we get:

$$\sigma_{ij} + \alpha\, p\, \delta_{ij} = 2\, G\, \varepsilon_{ij} + \lambda\, \varepsilon_{kk}\, \delta_{ij}\,. \tag{2.43}$$

It is clear from the above equation, that any strain of the matrix is due to the expression on the l.h.s., which is exactly the effective stress $(\tilde{\sigma}_{ij})$ defined by eq. (2.37).

Now, according to (2.43), we can easily rewrite the constitutive equations (2.35) in terms of the effective stress $(\tilde{\sigma}_{ij})$:

$$
\begin{aligned}
\tilde{\sigma}_{ij} &= 2\, G\, \varepsilon_{ij} + \lambda\, \varepsilon_{kk}\, \delta_{ij}\,, \\
\zeta &= \alpha\, \varepsilon_{kk} + \frac{1}{M}\, p\,.
\end{aligned}
\tag{2.44}
$$

The advantage of expressing the constitutive equations in terms of the effective stress consists in the fact that the equations thus become formally identical with conventional elasticity equations. In other words, Hooke's law for a poroelastic medium given by the first equation of (2.44) becomes the same as Hooke's law for an elastic medium, i.e.,

$$\sigma_{ij}^{*} = 2\, G^{*}\, \varepsilon_{ij} + \lambda^{*}\, \varepsilon_{kk}\, \delta_{ij}\,, \tag{2.45}$$

provided the effective stress $(\tilde{\sigma}_{ij})$ is replaced by the stress acting in the elastic medium (σ_{ij}^{*}), and the values of poroelastic parameters $(G,\ \lambda)$ are replaced by the values corresponding to the elastic medium $(G^{*},\ \lambda^{*})$.

2.3.2 Constitutive Equations of Linear Thermoporoelasticity

A straightforward extension of the constitutive equations (2.35) valid under isothermal conditions $(\delta\, T = 0)$ to a non-isothermal case can be performed, leading to the constitutive equations of linear thermoporoelasticity (Palciauskas and Domenico, 1982), (Charlez, 1991):

$$
\begin{aligned}
\sigma_{ij} &= 2\, G\, \varepsilon_{ij} + \lambda\, \varepsilon_{kk}\, \delta_{ij} - \alpha\, p\, \delta_{ij} - \left(\lambda + \frac{2\, G}{3}\right) \alpha_{b}\, T\, \delta_{ij}\,, \\
\zeta &= \alpha\, \varepsilon_{kk} + \frac{1}{M}\, p - (\alpha\, \alpha_{b} + \alpha_{m})\, T\,,
\end{aligned}
\tag{2.46}
$$

Here, α_{b} and α_{m} are thermal expansion coefficients (will be defined in the next section). We note again that the values of stress (σ_{ij}), pore fluid pressure (p), and temperature (T) in the above equations should be understood as the variations in the given quantities measured from the initial state (e.g., $T \equiv \delta\, T = T - T_{0}$). It is clear from

eq. (2.46) that the shear stresses ($i \neq j$) remain unaffected by the changes in temperature, thus the constitutive equations for the shear components of stress/strain fields are the same both under isothermal and non-isothermal conditions.

2.4 Poroelastic Parameters

In general, four independent poroelastic parameters appear in the seven constitutive equations of linear isothermal poroelasticity, provided the bulk material is isotropic. Thus, in comparison with the classical linear elasticity theory, two additional parameters are needed in the case of poroelasticity.

There are several different sets of the four poroelastic parameters found in the literature, as their choice depends on what is the most convenient to measure in the particular problem. Hence, the constitutive equations appear in many alternative forms. There is always at least one parameter in the complete set related to the shear deformation of the material, and the remaining (three or less) parameters are associated with the volumetric response. Algebraic relationships exist among the different poroelastic parameters, similarly as in the classical elasticity theory.

The poroelastic parameters can be grouped into 6 categories:

1. Lame's parameters.
2. Compressibilities.
3. Poisson's ratio.
4. Storage coefficients.
5. Pore fluid pressure buildup coefficients.
6. Hydraulic diffusivity.

Several parameters can be defined within each group, because the properties of various rock constituents (matrix/solid/fluid) can be measured under various conditions (e.g., drained/undrained conditions).

2.4.1 Lame's Parameters

Similarly as for classical elasticity, two Lame's parameters can be associated with a given poroelastic material under given conditions. Since there are two limiting deformation conditions to deal with in poroelasticity - the drained conditions ($p = 0$) and

the undrained conditions ($\zeta = 0$), the total number of Lame's parameters assigned to a given poroelastic material raises to four. However, only three of them are generally of different values.

In order to characterize the elastic properties of the drained poroelastic material, i.e., of the matrix, Lame's parameters are measured under drained conditions ($p = 0$). Their values are thus not influenced by the pore fluid, and they are called *drained Lame's parameters*. *Drained Lame's first parameter*, hereafter denoted by λ, has no physical interpretation (and therefore no special name), but can be expressed in terms of other poroelastic parameters, which already have a direct physical meaning (e.g., K and G, see the following text). *Drained Lame's second parameter*, hereafter denoted by G, can be physically interpreted as follows. When a linear poroelastic material is subjected to a pure shear deformation, in which the normal strain components are zero (i.e., $\varepsilon_{ij} = 0$ for $i = j$), the constitutive equations for the stress tensor components given by the first eq. of (2.35) are reduced to: $\sigma_{ij} = 2\,G\,\varepsilon_{ij}$. Thus, the poroelastic parameter denoted by G quantifies the rigidity of a poroelastic material subjected to a shear deformation. Unlike λ, it has a special name - it is commonly called *shear modulus*.

In order to characterize the elastic properties of the bulk poroelastic material (accounting for a possible pore fluid influence), Lame's parameters are measured under undrained conditions ($\zeta = 0$). They are called *undrained Lame's parameters* in this case. *Undrained Lame's first parameter* is hereafter denoted by λ_u, and its value in general differs from the value of λ, since the pore fluid pressure (p) affects the bulk elastic response to the volumetric deformation. *Undrained Lame's second parameter* is of the same value as the drained Lame's second parameter, i.e., the shear modulus (G), since the pore fluid pressure cannot produce any shearing strain in the isotropic bulk material and contribute to the shear deformation of the bulk material. Hence, the shear modulus (G) fully and unambiguously characterizes the poroelastic response of a given material to the shear deformation.

2.4.2 Compressibilities

The compressibility of a material quantifies the volume change of the material induced by a pressure exerted on the material. While for a non-porous material only a single compressibility exists, several compressibilities are associated with a given fluid-saturated porous material. This arises from the fact that the bulk material can be subjected to various pressures and various volume changes can be considered. The pressure can be the pore fluid pressure (p), the confining pressure (P_c), or the differential pressure (P_d). The volume can can be the bulk volume (V_b), the pore fluid volume (V_f), the volume of the pores (V_p), or the volume of the solid material (V_s). However, only two of the volumes are independent. Similarly, only two pressures can be varied independently. Thus, 4 basic compressibilities can be defined, while the other compressibilities are derivable from them. The compressibilities associated with the bulk volume (V_b), the pore volume (V_p), the pore fluid pressure (p), and the confining pressure (P_c) are defined as follows (Zimmerman et al., 1991):

$$\beta_{bP_c} \;=\; -\frac{1}{V_b^0}\frac{\partial V_b}{\partial P_c}\Big|_{p=0}\,, \tag{2.47}$$

$$\beta_{bp} \;=\; \frac{1}{V_b^0}\frac{\partial V_b}{\partial p}\Big|_{P_c=0}\,, \tag{2.48}$$

$$\beta_{pP_c} \;=\; -\frac{1}{V_p^0}\frac{\partial V_p}{\partial P_c}\Big|_{p=0}\,, \tag{2.49}$$

$$\beta_{pp} \;=\; \frac{1}{V_p^0}\frac{\partial V_p}{\partial p}\Big|_{P_c=0}\,. \tag{2.50}$$

Here, the first subscript on the l.h.s. denotes the volume which is being changed and the second subscript determines which pressure is varied. In the above definitions is assumed that the temperature is constant during the pressure variation process.

The compressibility defined by eq. (2.47) is called *drained bulk compressibility* (β_{bP_c}), and its inverse is called *drained bulk modulus*:

$$K \;=\; \frac{1}{\beta_{b\,P_c}}\,. \tag{2.51}$$

The drained bulk modulus (K) measures the bulk volume change (V_b) due to the applied confining pressure (P_c). Its value can be determined experimentally under drained conditions ($p = 0$). The drained bulk modulus actually expresses the compressibility

of the matrix, therefore it is sometimes called *bulk modulus of the matrix*. For brevity, the drained bulk compressibility (β_{bP_c}) will be hereafter denoted as β_b.

Under conditions of uniaxial strain (e.g., ε_{zz}) and constant normal stress in the strain direction ($\sigma_{zz} = 0$), the *uniaxial bulk compressibility* (also called *vertical compressibility*) is used:

$$\beta_b^v = -\frac{1}{V_b^0}\frac{\partial V_b}{\partial P_c}\Big|_{\sigma_{zz}=0,\,\varepsilon_{xx}=\varepsilon_{yy}=0}. \tag{2.52}$$

The compressibility defined by eq. (2.48) is called *bulk volume expansion coefficient*, or *poroelastic expansion coefficient*. Its inverse is often denoted by H, according to Biot (1941),

$$\beta_{b\,p} = \frac{1}{H}. \tag{2.53}$$

This poroelastic parameter has no equivalence in the classical elasticity. It measures the bulk volume change (V_b) due to the pore fluid pressure change (p), while the confining pressure is held constant $(P_c = 0)$. It is related to the drained bulk modulus (K) and to the Biot-Willis coefficient (α) as follows:

$$\beta_{b\,p} = \frac{\alpha}{K}. \tag{2.54}$$

Eq. (2.49) defines the *drained pore compressibility*. Hereafter, it will be denoted by β_p. Its inverse, K_p, is called *drained pore bulk modulus* and is related to the drained bulk modulus (K) as follows:

$$\frac{1}{K_p} = \frac{\alpha}{\phi_0\,K}. \tag{2.55}$$

The drained pore compressibility (β_p) measures the pore volume change (V_p) due to the applied confining pressure (P_c), while maintaining the pore fluid pressure constant $(p = 0)$.

The compressibility defined by eq. (2.50) is called *pore volume expansion coefficient*. It is related to the drained pore bulk modulus (K_p) according to

$$\beta_{p\,p} = \frac{\beta}{K_p}, \tag{2.56}$$

where β is the *effective stress coefficient for the pore volume*. It is defined in an analogous manner as the effective stress coefficient for the bulk volume, i.e., the Biot-Willis coefficient (α). The definition of α reads:

$$\alpha \;=\; 1 \;-\; \frac{K}{K'_s} \,. \tag{2.57}$$

Here, K'_s denotes the *unjacketed bulk modulus* and its inverse is called *unjacketed bulk compressibility*. The coefficient β is defined as

$$\beta \;=\; 1 \;-\; \frac{K_p}{K''_s} \,, \tag{2.58}$$

where K''_s denotes the *unjacketed pore modulus* and its inverse is called *unjacketed pore compressibility*. Sometimes it is denoted by K_ϕ instead of K''_s (Wang, H., 2000). The unjacketed bulk modulus (K'_s) and the unjacketed pore modulus (K''_s) are derivable from the 4 basic compressibilities defined by eqs. (2.47) - (2.50):

$$\frac{1}{K'_s} \;=\; \beta_{b\,P_c} \;-\; \beta_{b\,p} \,, \tag{2.59}$$

$$\frac{1}{K''_s} \;=\; \beta_{p\,P_c} \;-\; \beta_{p\,p} \,. \tag{2.60}$$

Alternatively, the unjacketed bulk compressibility ($1\,/\,K'_s$) and the unjacketed pore compressibility ($1\,/\,K''_s$) can be defined in the same way as the four basic compressibilities (2.47)-(2.50), provided the varied pressure is now the differential pressure (P_d) instead of the confining pressure (P_c) or of the pore fluid pressure (p). Then, the unjacketed bulk compressibility is given as

$$\frac{1}{K'_s} \;=\; -\,\frac{1}{V_b^0} \frac{\partial\, V_b}{\partial\, p} \,\Big|_{P_d = 0} \,, \tag{2.61}$$

and the unjacketed pore compressibility is

$$\frac{1}{K''_s} \;=\; -\,\frac{1}{V_p^0} \frac{\partial\, V_p}{\partial\, p} \,\Big|_{P_d = 0} \,. \tag{2.62}$$

Under conditions of an ideal porous material, the Eulerian porosity (ϕ) remains constant if the induced pore fluid pressure change $(\delta\, p)$ and the confining pressure change $(\delta\, P_c)$ are equal, i.e., if the differential pressure is held constant $(\delta\, P_d \;=\; 0)$. The definition of the Eulerian porosity (2.9) then implies:

$$\frac{V_p}{V_b} \;=\; \frac{V_p^0}{V_b^0} \,. \tag{2.63}$$

A simple algebra shows that the above equation can be rewritten as

$$\frac{\delta\, V_p}{V_p^0} \;=\; \frac{\delta\, V_b}{V_b^0} \,. \tag{2.64}$$

Then, from eqs. (2.61), (2.62) and (2.64) follows that the unjacketed bulk modulus (K'_s) and the unjacketed pore modulus (K''_s) are identical, i.e., $K'_s \;=\; K''_s$. This equality is

sometimes used as the definition of the ideal porous material (Detournay and Cheng, 1993). Experiments suggest that this equality is valid for most rocks and that a possible difference between the two moduli can be due to a presence of non-connected pores.

Let us define the *solid bulk compressibility* as follows:

$$\beta_s = -\frac{1}{V_s^0} \frac{\partial V_s}{\partial p} \mid_{P_d = 0}. \tag{2.65}$$

The inverse of β_s is called *solid bulk modulus*, hereafter denoted by K_s,

$$K_s = \frac{1}{\beta_s}. \tag{2.66}$$

From eqs. (2.21), (2.66) and (2.65) follows that the unjacketed bulk modulus (K_s') defined by eq. (2.59) can be identified as the solid bulk modulus (K_s), i.e., $K_s' = K_s$. This identity becomes quite obvious by realizing that a pore fluid pressure change (p) performed at constant differential pressure ($P_d = 0$) induces the bulk volume change (V_b) predominantly by acting on the solid phase of the rock, i.e., on the individual solid grains. The grain-to-grain stresses remain unchanged because the pore fluid pressure change counteracts the equal change in the confining pressure. However, the equality is satisfied only for the rocks composed of a single solid constituent and not for those composed of different minerals.

Altogether, the three moduli K_s'', K_s' and K_s are identical for an ideal porous material:

$$K_s'' = K_s' = K_s. \tag{2.67}$$

Then, the definition of Biot-Willis coefficient (α) can be rewritten as

$$\alpha = 1 - \frac{K}{K_s}, \tag{2.68}$$

or alternatively in terms of compressibilities instead of the moduli:

$$\alpha = 1 - \frac{\beta_s}{\beta_b}. \tag{2.69}$$

The above equation represents the most commonly used definition of the Biot-Willis coefficient (α).

In order to be complete, let us define the *pore fluid compressibility*:

$$\beta_f = -\frac{1}{V_f^0} \frac{\partial V_f}{\partial p}. \tag{2.70}$$

The inverse of β_f is called *pore fluid bulk modulus*, and denoted by K_f,

$$K_f \;=\; \frac{1}{\beta_f} \,. \tag{2.71}$$

An alternative definition of the pore fluid compressibility (β_f) in terms of the pore fluid density (ρ_f) is often used:

$$\beta_f \;=\; \frac{1}{\rho_f^0} \frac{\partial \rho_f}{\partial p} \,. \tag{2.72}$$

The last parameter which has to be defined here is the *undrained bulk modulus*, commonly denoted by K_u. It quantifies the volume change of the bulk material under undrained conditions $(\zeta = 0)$, and is defined as the ratio of the variation in the mean stress (σ) and the corresponding bulk volumetric strain (ε_{kk}),

$$K_u \;=\; \frac{\sigma}{\varepsilon_{kk}} \,\big|_{\,\zeta \,=\, 0} \,. \tag{2.73}$$

The undrained bulk modulus (K_u) is related to the drained bulk modulus (K) through *Biot-Gassmann equation*, which has the following form for an ideal porous material:

$$\frac{1}{K} - \frac{1}{K_u} \;=\; \frac{\left(\dfrac{1}{K} - \dfrac{1}{K_s} \right)^2}{\dfrac{1}{K} - \dfrac{1}{K_s} + \phi_0 \left(\dfrac{1}{K_f} - \dfrac{1}{K_s} \right)} \,. \tag{2.74}$$

2.4.3 Poisson's Ratio

Poisson's ratio is defined as the ratio of *transverse strains* to *axial strains*, induced in the bulk material under uniaxial loading (σ_{ii}) (taken with the minus sign). The axial and the transverse strains are, respectively, the longitudinal strains in the direction of the applied stress (i), and in the perpendicular direction (j). There are two different values of Poisson's ratio associated with a given poroelastic material.

In the case of loading performed at a constant pore fluid pressure $(p = 0)$, i.e., under drained conditions, the *drained Poisson's ratio* is defined:

$$\nu \;=\; - \frac{\varepsilon_{jj}}{\varepsilon_{ii}} \,\big|_{\,(\sigma_{jj} \,=\, 0 \,,\, p \,=\, 0)} \quad ; \;\; (i \,\neq\, j) \,. \tag{2.75}$$

Analogously, the *undrained Poisson's ratio* is defined under undrained conditions $(\zeta = 0)$ as follows:

$$\nu_u \;=\; - \frac{\varepsilon_{jj}}{\varepsilon_{ii}} \,\big|_{\,(\sigma_{jj} \,=\, 0 \,,\, \zeta \,=\, 0)} \quad ; \;\; (i \,\neq\, j) \,. \tag{2.76}$$

Similarly as in the classical elasticity, the relations between the drained and undrained Poisson's ratios and another commonly used poroelastic parameters can be derived.

The following two relations between the Poisson's ratios and the drained bulk modulus (K), the undrained bulk modulus (K_u), and the shear modulus (G) are often used:

$$\nu = \frac{3\,K \;-\; 2\,G}{2\,(\,3\,K \;+\; G\,)}\,, \tag{2.77}$$

$$\nu_u = \frac{3\,K_u \;-\; 2\,G}{2\,(\,3\,K_u \;+\; G\,)}\,. \tag{2.78}$$

2.4.4 Storage Coefficients

Storage coefficients quantify the change in the pore fluid volume per unit bulk volume induced by a pore fluid pressure change. They are defined as the ratio of the variation of fluid content (ζ) and the pore fluid pressure change ($\delta\,p$) under various conditions.

The storage coefficient measured under the constant mean normal stress ($\sigma = 0$) is called *unconstrained specific storage coefficient*, and is defined as

$$S_\sigma = \frac{\zeta}{\delta\,p}\,\Big|_{\,\sigma = 0}\,. \tag{2.79}$$

The unconstrained specific storage coefficient is a reciprocal of Biot's coefficient denoted by R (Biot, 1941):

$$S_\sigma = \frac{1}{R}\,. \tag{2.80}$$

The following expression for the unconstrained specific storage coefficient (S_σ) in terms of various compressibilities, valid for an ideal porous material, is often used:

$$S_\sigma = (\,\beta_b \;-\; \beta_s\,) \;+\; n\,(\,\beta_f \;-\; \beta_s\,)\,. \tag{2.81}$$

The second specific storage coefficient, called *constrained specific storage coefficient*, is measured under conditions of zero volumetric strain ($\varepsilon = 0$) and its definition reads:

$$S_\varepsilon = \frac{\zeta}{\delta\,p}\,\Big|_{\,\varepsilon = 0}\,. \tag{2.82}$$

Also this coefficient was introduced by Biot, who denoted its inverse as Q. However, the reciprocal of S_ε is mostly denoted by M instead of Q, and called *Biot modulus* (Detournay and Cheng, 1993):

$$M = \frac{1}{S_\varepsilon}\,. \tag{2.83}$$

Under conditions of constant normal stress ($\sigma_{zz} = 0$) and uniaxial strain in the normal direction ($\varepsilon_{zz} \neq 0$), it is useful to define the *uniaxial specific storage coefficient*:

$$S_a = \frac{\zeta}{\delta p} \,\big|\, \sigma_{zz} = 0 \,,\, \varepsilon_{xx} = \varepsilon_{yy} = 0 \,. \qquad (2.84)$$

Substituting the constraints $\sigma_{zz} = 0$ and $\varepsilon_{xx} = \varepsilon_{yy} = 0$ into the constitutive equations for the normal stress components, given by the first three equations of (2.36), we obtain:

$$\sigma_{kk} = -4\,\eta\,p\,. \qquad (2.85)$$

Here, the dimensionless variable denoted by η is called *poroelastic stress coefficient* (Detournay and Cheng, 1993). It can be expressed as

$$\eta = \frac{1 - 2\nu}{2(1 - \nu)}\,\alpha\,. \qquad (2.86)$$

The uniaxial specific storage coefficient (S_a) is related to the uniaxial bulk compressibility (β_b^v), the pore fluid compressibility (β_f), and the drained pore compressibility (β_p) as follows:

$$S_a = \alpha^2\,\beta_b^v + n\,(\beta_f - \beta_p)\,. \qquad (2.87)$$

Another specific storage coefficient, introduced by Biot and Willis (1957) under the term *coefficient of fluid content*, is sometimes used. It is defined as the ratio of the variation of fluid content (ζ) and the pore fluid pressure change (δp), under conditions of constant differential pressure (P_d):

$$S_\gamma = \frac{\zeta}{\delta p}\,\big|\, P_d = 0 \,. \qquad (2.88)$$

It is also called *unjacketed specific storage coefficient* (Wang, H., 2000).

2.4.5 Pore Fluid Pressure Buildup Coefficients

The most important coefficient of this group, and actually one of the most important among all poroelastic parameters, is *Skempton coefficient*. It is defined for undrained conditions $(\zeta = 0)$, as the ratio of the pore fluid pressure (p) induced by the applied mean normal stress (σ), and the applied mean normal stress (σ):

$$B = -\frac{p}{\sigma}\,\big|\, \zeta = 0 \,. \qquad (2.89)$$

Note that the minus sign follows from the sign convention, in which the compressive stresses are taken as negative, while the pore fluid pressure is always positive.

A useful expression for Skempton coefficient in terms of fundamental parameters can be obtained by taking the following steps:

1. First, the last constitutive relation given in eq. (2.36) is written under undrained conditions ($\zeta = 0$):

$$\alpha \, \varepsilon_{kk} \;\; = \;\; - \, \frac{p}{M} \; . \tag{2.90}$$

2. Now, using the definition of the mean normal stress (σ), given by eq. (2.25), and the constitutive equations for the normal stress components, given in eq. (2.36), the volumetric strain can be expressed as

$$\varepsilon_{kk} \;\; = \;\; \frac{3 \, (\, \sigma \, + \, \alpha \, p \,)}{2 \, G \, + \, 3 \, \lambda} \; . \tag{2.91}$$

3. A substitution of eq. (2.91) into eq. (2.90), using the definition of Skempton coefficient (2.89), leads to the expression:

$$B \;\; = \;\; \frac{3}{3 \, \alpha \; + \; \dfrac{2 \, G \, + \, 3 \, \lambda}{\alpha \, M}} \; . \tag{2.92}$$

4. Finally, the following relations between poroelastic parameters are used in order to get the expression for B in terms of Poisson's ratios,

$$M \;\; = \;\; \frac{2 \, G \, (\, \nu_u \, - \, \nu \,)}{\alpha^2 \, (\, 1 \, - \, 2 \, \nu_u \,) \, (\, 1 \, - \, 2 \, \nu \,)} \; , \tag{2.93}$$

$$\frac{\lambda}{G} \;\; = \;\; \frac{2 \, \nu}{1 \, - \, 2 \, \nu} \; . \tag{2.94}$$

5. After the substitution of eqs. (2.93) and (2.94) into eq. (2.92), we obtain the desired expression for Skempton coefficient (B) in terms of the drained and undrained Poisson's ratios (ν, ν_u) and Biot-Willis coefficient (α):

$$B \;\; = \;\; \frac{3 \, (\, \nu_u \, - \, \nu \,)}{\alpha \, (\, 1 \, + \, \nu_u \,) \, (\, 1 \, - \, 2 \, \nu \,)} \; . \tag{2.95}$$

Another often used expression for Skempton coefficient in terms of the moduli K, K_u, K_s can be obtained from eq. (2.95) by using eqs. (2.77), (2.78), (2.57), and the following relation between the shear modulus (G), drained bulk modulus (K), and the drained Poisson's ratio (ν),

$$G \;\; = \;\; \frac{3 \, K}{2} \, \frac{(\, 1 \, - \, 2 \, \nu \,)}{(\, 1 \, + \, \nu \,)} \; . \tag{2.96}$$

Then, the expression for Skempton coefficient reads:

$$B \;\; = \;\; \frac{1 \, - \, \dfrac{K}{K_u}}{1 \, - \, \dfrac{K}{K_s}} \; . \tag{2.97}$$

The last relation yields the Biot-Gassmann equation (2.74) in terms of Skempton coefficient:

$$B = \frac{\left(\dfrac{1}{K} - \dfrac{1}{K_s} \right)}{\dfrac{1}{K} - \dfrac{1}{K_s} + \phi_0 \left(\dfrac{1}{K_f} - \dfrac{1}{K_s} \right)} .$$
(2.98)

2.4.6 Hydraulic Diffusivity

The hydraulic diffusivity measures the rate at which a disturbance in the pore fluid pressure propagates through the porous medium (determined by its *permeability* (k), porosity (ϕ), and drained pore compressibility (β_p)) saturated with the pore fluid (characterized by its density (ρ_f), compressibility (β_f), and *dynamic fluid viscosity* (μ)). Its definition reads:

$$\alpha_{hy} = \frac{k}{\mu \, S_\sigma} .$$
(2.99)

The ratio k/μ is called *mobility coefficient.*

Under conditions of effectively incompressible solid phase ($\beta_s = 0$), eq. (2.81) for the unconstrained specific storage coefficient (S_σ) is simplified to

$$S_\sigma = \beta_b + n\,\beta_f ,$$
(2.100)

and the hydraulic diffusivity is given as follows:

$$\alpha_{hy} = \frac{k}{\mu} \frac{1}{(\beta_b + n\,\beta_f)} .$$
(2.101)

The permeability (k) appearing in eq. (2.99) is a property intrinsic to the drained porous material, i.e., to the matrix. Therefore it is also referred to as *intrinsic permeability.* It measures the geometry of interconnected pores and is proportional to the ease with which a pore fluid flows through the matrix. Various expressions for the permeability, as a function of another material parameters have been proposed. For example, its dependency on the Eulerian porosity (ϕ) is given by a widely used Kozeny-Carman relation,

$$k = \frac{\phi^3}{c \, A_p^2} ,$$
(2.102)

where c is *Kozeny constant* accounting for the geometry of the pore space (e.g., $c = 2$ for cylindrical capillaries), and A_p denotes the net area of pores per unit bulk volume.

2.4.7 Parameters of Linear Thermoporoelasticity

Two additional material parameters are present in constitutive equations of linear thermoporoelasticity in comparison with the constitutive equations of isothermal linear poroelasticity. Therefore, six independent parameters now appear in the constitutive equations. The two additional parameters are taken from the group called *thermal expansion coefficients*. Furthermore, the parameters called *thermal diffusivity*, *volumetric heat capacity*, and *thermal conductivity* must be introduced, since they appear in the governing equations of thermoporoelasticity.

Thermal Expansion Coefficients. Thermal expansion coefficients, called also *thermal expansivities*, measure the volumetric deformation of the material due to a temperature change. For a fluid-saturated porous material, variations in four different volumes can be considered, namely the bulk volume (V_b), the pore fluid volume (V_f), the volume of the pores (V_p), and the solid material volume (V_s). A given temperature change (T) generally leads to a different change in each of the volumes mentioned above, hence four different thermal expansivities must be introduced:

- The *drained bulk thermal expansivity*, denoted as α_b, is defined as

$$\alpha_b \quad = \quad \frac{1}{V_b^0} \frac{\partial V_b}{\partial T} \big|_{P_c,\, p\, =\, 0} \cdot \tag{2.103}$$

- The *pore fluid thermal expansivity*, denoted as α_f, is defined as

$$\alpha_f \quad = \quad \frac{1}{V_f^0} \frac{\partial V_f}{\partial T} \big|_{P_c,\, p\, =\, 0} \cdot \tag{2.104}$$

- The *solid thermal expansivity*, denoted as α_s, is defined as

$$\alpha_s \quad = \quad \frac{1}{V_s^0} \frac{\partial V_s}{\partial T} \big|_{P_c,\, p\, =\, 0} \cdot \tag{2.105}$$

Under conditions of an ideal porous material, for which eq. (2.21) holds, the solid thermal expansivity (α_s) and the drained bulk thermal expansivity (α_b) are actually identical, i.e.,

$$\alpha_s \quad = \quad \alpha_b \cdot \tag{2.106}$$

- The *pore volume thermal expansivity*, denoted as α_p, is defined as

$$\alpha_p \quad = \quad \frac{1}{V_p^0} \frac{\partial V_p}{\partial T} \big|_{P_c,\, p\, =\, 0} \cdot \tag{2.107}$$

Alternatively, the above definition can be rewritten in terms of the Lagrangian porosity (n) according to its definition (2.8):

$$\alpha_p \;=\; \frac{1}{n_0} \frac{\partial\, n}{\partial\, T} \Big|_{P_c,\, p\, =\, 0} . \tag{2.108}$$

Similarly as for the solid thermal expansivity (α_s), the pore volume thermal expansivity (α_p) is identical to the drained bulk thermal expansivity (α_b) under conditions of an ideal porous material,

$$\alpha_p \;=\; \alpha_b , \tag{2.109}$$

as clearly follows from eq. (2.64).

In addition to the four thermal expansivities given above, the *undrained bulk thermal expansivity*, denoted as α_u, and the *fluid mass content thermal expansivity*, denoted as α_m, are sometimes used. The fluid mass content thermal expansivity (α_m) is defined as

$$\alpha_m \;=\; \frac{1}{m_f^0} \frac{\partial\, m_f}{\partial\, T} \Big|_{P_c,\, p\, =\, 0} , \tag{2.110}$$

and the undrained bulk thermal expansivity (α_u) is introduced through the following relation valid for an ideal porous material:

$$\alpha_u \;=\; \alpha_s \;+\; B\, n_0 \, (\, \alpha_f \;-\; \alpha_s \,) . \tag{2.111}$$

Another useful relation between thermal expansivities reads:

$$\alpha_m \;=\; \alpha_p \;-\; \alpha_f . \tag{2.112}$$

Effective Volumetric Heat Capacity. The *effective volumetric heat capacity* of a fluid-saturated porous material is the amount of heat required to change the temperature of a unit bulk volume by one degree. It is defined as (Bear, 1988)

$$c_e \;=\; \rho_b\, c_b \;=\; \phi\, \rho_f\, c_f \;+\; (\, 1 \;-\; \phi \,)\, \rho_s\, c_s , \tag{2.113}$$

where ρ_b and ρ_s are densities of the bulk and of the solid material, respectively, and c_b, c_f, and c_s denote respectively the *specific heat capacity* of the bulk material, of the pore fluid, and of the solid material. The specific heat capacity of a given material is defined as the amount of heat required to change the temperature of a unit mass of the material by one degree.

Thermal Diffusivity. The thermal diffusivity, similarly as the hydraulic diffusivity (α_{hy}), is the measure of the rate at which a disturbance in temperature propagates through a fluid-saturated porous material. For a macroscopically homogeneous bulk material it is defined as:

$$\alpha_{th} = \frac{k_{th}}{c_e} . \tag{2.114}$$

Here, k_{th} denotes the *effective thermal conductivity* of the bulk material. The effective thermal conductivity of the bulk material (k_{th}) measures the bulk material ability to conduct heat and is defined as follows (Mase and Smith, 1985), (Bejan et al., 2004):

$$k_{th} = k_f^{\phi} \, k_s^{1-\phi} . \tag{2.115}$$

Here, k_f is the *pore fluid thermal conductivity*, and k_s is the *solid thermal conductivity*.

2.5 Governing Equations

There are eleven unknowns generally appearing in the governing equations associated with a problem of linear isothermal poroelasticity:

- Six stress components (σ_{ij}).
- Three displacement components (u_i).
- Pore fluid pressure (p).
- Variation of fluid content (ζ).

The strain components (ε_{ij}) are not involved in the list, since they can be expressed in terms of the displacement components (u_i), according to the strain tensor definition (2.1).

The governing equations needed to determine the eleven unknowns are derived by considering the following physical laws:

- Constitutive equations (consisting of seven equations for the bulk material).
- Force balance equations (consisting of three equations for the bulk material).
- Fluid continuity equation (consisting of one equation for the pore fluid).
- Darcy's law, i.e., the fluid diffusion law (consisting of three equations for the pore fluid).

Altogether, 14 scalar equations are considered. Darcy's law can be combined with the fluid continuity equation, which lead to only one equation for the pore fluid called *fluid diffusivity equation.* Hence, the total number of the scalar equations is reduced to eleven, in agreement with the number of unknowns. The final set of eleven governing equations is composed of

- seven constitutive equations,
- three force balance equations,
- one fluid diffusivity equation.

In general, the governing equations are fully coupled and therefore difficult to solve. Under certain conditions, however, some of them become uncoupled and can be solved independently of the others.

2.5.1 Force Balance Equations

According to the assumption of quasi-static deformation processes, the REV is maintained in the state of internal force equilibrium. It means an instantaneous adjustment of the pore fluid pressure (p) and displacements (u_i) within the REV to the applied stress change, in order to preserve the equilibrium state. The equilibrium state must involve both rotational and translational force balances.

From the rotational force balance of the REV follows the symmetry of the total stress tensor, i.e., $\sigma_{xy} = \sigma_{yx}$, $\sigma_{yz} = \sigma_{zy}$, $\sigma_{zx} = \sigma_{xz}$. Written in a concise form,

$$\sigma_{ij} = \sigma_{ji} . \tag{2.116}$$

The translational force balance equations are obtained from the fact that the net force on the REV must be zero in every Cartesian direction (x, y, z). Hence the three force balance equations are obtained in the form

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0 , \tag{2.117}$$

where f_i is the i-th component of the *body force per unit bulk volume.*

In many practical applications the body force term (f_i) can be neglected and the force balance equations read:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0 . \tag{2.118}$$

Displacement Formulation of Force Balance Equations. The *displacement formulation of the force balance equations* (called also *Navier-type equations*, according to Detournay and Cheng (1993)) can be formulated in two ways, since either the pore fluid pressure (p) or the variation of fluid content (ζ) can be used. The displacement formulation in terms of the pore fluid pressure (p) is obtained by taking the following two steps:

1. The six constitutive equations for the stress tensor components (2.36) are substituted into the force balance equations (2.117).

2. The strain tensor definition (2.1) is applied, leading to the following three force balance equations,

$$G \, \nabla^2 u_i \; + \; (\, G \; + \; \lambda \,) \, \frac{\partial^{\,2} u_k}{\partial \, x_i \, x_k} \;\; = \;\; \alpha \, \frac{\partial \, p}{\partial \, x_i} \; - \; f_i \,, \qquad\qquad (2.119)$$

where $i \; \in \; \{ 1, \; 2, \; 3 \,\}$.

The only term responsible for the difference between the force balance equations (2.119) and their equivalent in the classical elasticity, is the first term on the r.h.s. containing the pore fluid pressure gradient. This fact, together with the presence of the mean normal stress/the volumetric strain term in the fluid diffusivity equation, makes the theory of poroelasticity more complicated in comparison with the elasticity theory, since the governing equations of poroelasticity are fully coupled in general.

The displacement formulation of the force balance equations in terms of the variation of fluid content (ζ) can be obtained from the displacement formulation in terms of the pore fluid (p). Differentiating the last constitutive relation in eq. (2.36) with respect to x_i, and substituting the obtained expression for the pore fluid pressure gradient into eq. (2.119), the desired force balance equations are

$$G \, \nabla^2 u_i \; + \; (\, G \; + \; \lambda_u \,) \, \frac{\partial^{\,2} u_k}{\partial \, x_i \, x_k} \;\; = \;\; \alpha \, M \, \frac{\partial \, \zeta}{\partial \, x_i} \; - \; f_i \,. \qquad\qquad (2.120)$$

In the above derivation, we used the following relations between the poroelastic parameters:

$$M \, \alpha \;\; = \;\; B \, K_u \,, \qquad\qquad (2.121)$$

$$B \, K_u \, \alpha \;\; = \;\; \lambda_u \; - \; \lambda \,. \qquad\qquad (2.122)$$

Stress Formulation of Force Balance Equations. The *stress formulation of force balance equations*, called also *Beltrami-Michell equations*, is derivable from *strain compatibility equations*. The strain compatibility equations are six second order partial differential equations, obtained by second spatial differentiation of the strain tensor components defined by (2.1):

$$
\begin{aligned}
2 \, \frac{\partial^2 \varepsilon_{xy}}{\partial x \, \partial y} &= \frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} \,, \\
2 \, \frac{\partial^2 \varepsilon_{yz}}{\partial y \, \partial z} &= \frac{\partial^2 \varepsilon_{yy}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} \,, \\
2 \, \frac{\partial^2 \varepsilon_{zx}}{\partial z \, \partial x} &= \frac{\partial^2 \varepsilon_{zz}}{\partial x^2} + \frac{\partial^2 \varepsilon_{xx}}{\partial z^2} \,, \\
\frac{\partial^2 \varepsilon_{xx}}{\partial y \, \partial z} &= \frac{\partial}{\partial x} \left(-\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{xz}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z} \right) \,, \\
\frac{\partial^2 \varepsilon_{yy}}{\partial z \, \partial x} &= \frac{\partial}{\partial y} \left(\frac{\partial \varepsilon_{yz}}{\partial x} - \frac{\partial \varepsilon_{zx}}{\partial y} + \frac{\partial \varepsilon_{yx}}{\partial z} \right) \,, \\
\frac{\partial^2 \varepsilon_{zz}}{\partial x \, \partial y} &= \frac{\partial}{\partial z} \left(\frac{\partial \varepsilon_{zy}}{\partial x} + \frac{\partial \varepsilon_{zx}}{\partial y} - \frac{\partial \varepsilon_{xy}}{\partial z} \right) \,.
\end{aligned}
\tag{2.123}
$$

After the substitution of the six constitutive equations for stress tensor components, given in eq. (2.36), reformulated to give expressions for the strain components (ε_{ij}), and of the force balance equations (2.117) into the strain compatibility equations (2.123), we obtain the Beltrami-Michell equations:

$$
\begin{aligned}
\nabla^2 \sigma_{ij} + \frac{1}{1 + \nu} \frac{\partial^2 \sigma_{kk}}{\partial x_i \, x_j} + 2 \, \eta \left(\frac{1 - \nu}{1 + \nu} \frac{\partial^2 p}{\partial x_i \, x_j} + \delta_{ij} \, \nabla^2 p \right) &= \\
= -\frac{\nu}{1 - \nu} \, \delta_{ij} \, \vec{\nabla} \cdot \vec{f} - \frac{\partial f_i}{\partial x_j} - \frac{\partial f_j}{\partial x_i} \,. &
\end{aligned}
\tag{2.124}
$$

Here, the following relation was used,

$$
\nabla^2 \left(\sigma_{kk} + 4 \, \eta \, p \right) = -\frac{1 + \nu}{1 - \nu} \, \vec{\nabla} \cdot \vec{f},
\tag{2.125}
$$

which can be derived by summing the three force balance equations in the displacement formulation, and substituting for the displacement (u_i) according to the constitutive equations (2.36) and the strain tensor definition (2.1).

2.5.2 Fluid Diffusivity Equation

The fluid diffusivity equation is the governing equation for the pore fluid. It is derived by combining the *fluid continuity equation* with *Darcy's law*, taking into account the

constitutive relation for the pore fluid variable. Depending on the choice of the pore fluid variable, two different formulations of the fluid diffusivity equation are possible:

- The fluid diffusivity equation for the pore fluid pressure (p).
- The fluid diffusivity equation for the variation of fluid content (ζ).

Each of these formulations can be further expressed either in terms of the mean normal stress (σ) or in terms of the volumetric strain (ε). Using the variation of fluid content (ζ) as the pore fluid variable, the fluid diffusivity equation can be expressed in a very advantageous manner, without terms containing stress, strain or displacement, thus it becomes uncoupled.

Darcy's Law. Darcy's law represents the momentum balance equation for a macroscopic fluid flow through a porous material, induced by a pressure gradient. It is a phenomenological law proposed by Darcy in 1856 on the basis of his experiments with water flows through a vertical column of sand. The term macroscopic refers to the scale, sometimes called *Darcy scale*, which is much larger than the scale of the pores and much smaller than the scale of the macroscopic flow. Darcy's law in its simplest, linear formulation is given as

$$\vec{q} \;=\; -\,\frac{k}{\mu}\,\vec{\nabla}\,p\,, \tag{2.126}$$

where $\vec{q}$ is the so-called *Darcy's velocity*. Darcy's law is sometimes called *fluid diffusion law*.

Darcy's velocity ($\vec{q}$) can be further expressed as

$$\vec{q} \;=\; \phi\,\vec{v}\,, \tag{2.127}$$

where $\vec{v}$ is the relative velocity between the pore fluid and the solid material, called *seepage velocity*. It is defined as the difference between the *pore fluid velocity*, denoted by $\vec{v}_f$, and the *solid velocity*, denoted by $\vec{v}_s$:

$$\vec{v} \;=\; \vec{v}_f \;-\; \vec{v}_s\,. \tag{2.128}$$

According to (2.128), Darcy's velocity (2.127) is given as

$$\vec{q} \;=\; \phi\,(\,\vec{v}_f \;-\; \vec{v}_s\,)\,. \tag{2.129}$$

Darcy's velocity is the macroscopic volumetric flow velocity per unit area of the porous material, unlike the relative fluid to solid velocity in the individual pores, i.e., the seepage velocity ($\vec{v}$).

The applicability of the classical Darcy's law (2.126) is constrained by the following assumptions:

1. Body forces are neglected.

2. Advective inertial forces arising at high flow velocities are omitted. It means that either sufficiently small pressure gradients or sufficiently small flow rates are required in order to (2.126) being valid. The latter assumption is usually quantified by introducing the Reynolds number (Re) as the characteristic parameter of the flow. It is defined as

$$Re = \rho_f \frac{v\,l}{\mu}\,, \tag{2.130}$$

where l is the characteristic pore size. Above the critical value of Re the classical Darcy's law is no longer valid. The critical value of Reynolds number ranges from 0.1 to 75, although the upper bound is still a matter of research (Wang, L., 2000).

3. The pore fluid is incompressible. According to latest theoretical considerations (Wang, L., 2000), the pore fluid incompressibility could be responsible for the vanishing of the advective inertial term in Darcy's law, under conditions of a sufficiently small Reynolds number. This theoretical result has been proven for isotropic porous materials and one-dimensional flows. It is further conditioned by either a zero, a steady, or a rigid-body rotation flow. Nevertheless, we do not adopt neither of these assumptions, nor that of the pore fluid incompressibility, since they are still a matter of research.

4. The non-linear term accounting for viscous boundary layer effects (Vafai, 2005) is omitted. The theory suggests that these effects could become significant at intermediate Reynolds numbers.

Fluid Continuity Equation. The *fluid continuity equation* follows from the mass conservation principle applied to the pore fluid filling the interconnected pores of the matrix. Its general form in a Lagrangian reference frame with respect to the initial configuration of the solid matrix reads (Charlez, 1991), (Coussy, 2004):

$$\frac{\partial\, m_f}{\partial\, t}\ +\ \vec{\nabla}\, [\, m_f\, \vec{v}\,]\ =\ 0\ . \tag{2.131}$$

Hereafter, angular braces are used to enclose the terms on which a mathematical operator is applied.

We adopt the Lagrangian approach (not the Eulerian) in order to account for the coupled behavior between the pore fluid and the solid material during a deformation process. In the Lagrangian reference frame, both the pore fluid and the solid material motions refer to the initial solid configuration, thus can be easily related to each other. The convenience of the Lagrangian description results from the fact that also the strain tensor is defined with respect to the initial configuration. Quantities in the Lagrangian description are functions of time and of the position vector of the solid in the initial state. If the Eulerian approach was used, the quantities were functions of time and of the position vector of the solid in the actual state (or that of the fluid, which is the same in the Eulerian reference frame). The Eulerian description does not involve the initial configuration. Both the Lagrangian and Eulerian approaches result in identical final equations within the linear framework. We make a distinction between them, because we will consider also some non-linear equations.

According to the fluid mass content (m_f) definition (2.7), the fluid continuity equation (2.131) can be rewritten in the following way:

$$\frac{\partial}{\partial\, t}\, (\, \rho_f\, n\,)\ +\ \vec{\nabla}\, [\, \rho_f\, n\, \vec{v}\,]\ =\ 0\ . \tag{2.132}$$

Alternatively, the above equation can be written in terms of Darcy's velocity ($\vec{q}$). For this purpose, Darcy's velocity must be defined appropriately for the Lagrangian description, i.e., with respect to the initial configuration of the solid matrix:

$$\vec{q}^{\,L}\ =\ \vec{q}\, J\ =\ \phi\, \vec{v}\, J\ =\ n\, \vec{v}\ . \tag{2.133}$$

Eq. (2.133) is obtained by multiplying Darcy's velocity ($\vec{q}$), defined in eq. (2.127), by the Jacobian of transformation (J), introduced by eq. (2.14). Then the fluid continuity equation (2.132) in terms of Darcy's velocity ($\vec{q}^{\,L}$) reads:

$$\frac{\partial}{\partial\, t}\, (\, \rho_f\, n\,)\ +\ \vec{\nabla}\, [\, \rho_f\, \vec{q}^{\,L}\,]\ =\ 0\ . \tag{2.134}$$

For brevity, the superscript L will be omitted in the remaining discussion.

After some mathematical manipulations, the fluid continuity equation (2.134) can be further modified to take the following form under isothermal conditions:

44

$$\frac{\partial\, n}{\partial\, t} \; + \; n_0\, \beta_f\, \frac{\partial\, p}{\partial\, t} \; + \; \vec{\nabla}\cdot\vec{q} \; + \; \beta_f\, \vec{q}\cdot\vec{\nabla}\, p \;\; = \;\; 0 \,. \tag{2.135}$$

We do not perform a derivation of the above equation here, since it can be easily obtained from the general non-linear fluid diffusivity equation of thermoporoelasticity (2.159), a detailed derivation of which is given in the next subsection, by omitting the terms containing temperature variations, and taking into account Darcy's law (2.126).

The one-dimensional form of eq. (2.135) is often used (one deals with a one-directional fluid flow),

$$\frac{\partial\, n}{\partial\, t} \; + \; n_0\, \beta_f\, \frac{\partial\, p}{\partial\, t} \; + \; \frac{\partial\, q_x}{\partial\, x} \; + \; \beta_f\, q_x\, \frac{\partial\, p}{\partial\, x} \;\; = \;\; 0 \,, \tag{2.136}$$

where

$$\vec{q} \;\; = \;\; [\, q_x \,,\; 0 \,,\; 0 \,] \,. \tag{2.137}$$

The fluid continuity equation derived so far (2.136) is the general non-linear equation. However, we are interested in the linear form of the fluid continuity equation when working with linear poroelasticity. The most straightforward way to obtain it involves the following steps:

- First, the time derivative of eq. (2.23) is performed, leading to the relation:

$$\frac{\partial\, \zeta}{\partial\, t} \;\; = \;\; -\, \phi_0\, \frac{\partial}{\partial\, t}\left(\, \varepsilon_{kk}^f \; - \; \varepsilon_{kk}^s \,\right) \,. \tag{2.138}$$

- Following the strain tensor definition (2.1), eq. (2.138) can be expressed in terms of the *solid material displacement*, denoted by $\vec{u}_s$, and the *pore fluid displacement*, denoted by $\vec{u}_f$,

$$\frac{\partial\, \zeta}{\partial\, t} \;\; = \;\; -\, \phi_0\, \frac{\partial}{\partial\, t}\left\{\, \vec{\nabla}\cdot(\, \vec{u}_f \; - \; \vec{u}_s \,) \,\right\} \,. \tag{2.139}$$

- Finally, the basic relations between displacements and velocities, both of the solid material and of the pore fluid are used,

$$\vec{v}_s \;\; = \;\; \frac{\partial\, \vec{u}_s}{\partial\, t} \,, \tag{2.140}$$

$$\vec{v}_f \;\; = \;\; \frac{\partial\, \vec{u}_f}{\partial\, t} \,. \tag{2.141}$$

- After the substitution of eqs. (2.140) and (2.141) into eq. (2.139), and using the seepage velocity ($\vec{v}$) definition (2.128), the desired *linear form of the fluid continuity equation* is obtained:

$$\frac{\partial\, \zeta}{\partial\, t} \;\; = \;\; -\, \phi_0\, \vec{\nabla}\cdot\vec{v} \,. \tag{2.142}$$

45

The fluid diffusivity equation of linear poroelasticity (both for the pore fluid pressure and for the variation of fluid content) can be easily derived by using the linearized fluid continuity equation (2.142). However, we start the derivation with the general non-linear fluid continuity equation (2.135), in order to provide a better understanding of the origin of the final linear fluid diffusivity equation of poroelasticity. Moreover, the general non-linear equations will be used in the next chapter.

Fluid Diffusivity Equation for the Pore Fluid Pressure. The fluid diffusivity equation for the pore fluid pressure in its general non-linear form is obtained by substituting Darcy's law (2.126) into the fluid continuity equation (2.135):

$$\frac{\partial n}{\partial t} \;+\; n_0\,\beta_f\,\frac{\partial p}{\partial t} \;-\; \vec{\nabla}\left[\frac{k}{\mu}\,\vec{\nabla}\,p\right] \;-\; \frac{k}{\mu}\,\beta_f\left(\vec{\nabla}\,p\right)^2 \;=\; 0\,. \tag{2.143}$$

The last term on the l.h.s. is non-linear, hence can be omitted within the linear framework. Moreover, in order to avoid the non-linearity in the third term on the l.h.s., the assumption of spatial homogeneity of the material parameters k and μ is adopted. Then the linear fluid diffusivity equation for the pore fluid pressure reads:

$$\frac{\partial n}{\partial t} \;+\; n_0\,\beta_f\,\frac{\partial p}{\partial t} \;-\; \frac{k}{\mu}\,\Delta\,p \;=\; 0\,. \tag{2.144}$$

Here, Δ denotes the Laplacian ($\Delta \equiv \nabla^2$). Under conditions of an ideal porous material, the validity of the following relations can be demonstrated (the second equality is valid after the linearization):

$$\frac{\partial n}{\partial t} \;=\; \phi_0\,\dot{\varepsilon}_{kk}^s\,, \tag{2.145}$$

$$n_0\,\beta_f\,\frac{\partial p}{\partial t} \;=\; -\,n_0\,\frac{1}{V_f^0}\,\frac{\partial V_f}{\partial t} \;=\; -\,\phi_0\,\dot{\varepsilon}_{kk}^f\,. \tag{2.146}$$

(Hereafter, the dot symbol stands for the time derivative.) According to the relations given above, eq. (2.144) can be rewritten in terms of the variation of fluid content (ζ),

$$\frac{\partial \zeta}{\partial t} \;-\; \frac{k}{\mu}\,\Delta\,p \;=\; 0\,, \tag{2.147}$$

where the expression for the time derivative of ζ given by eq. (2.138) was used. Note that the same equation would be obtained straightforwardly by combining Darcy's law (2.126) with the linearized fluid continuity equation for an ideal porous material (2.142).

Substituting for ζ in eq. (2.147) according to the last constitutive relation given in eq. (2.36) leads to

46

$$\alpha \, \frac{\partial \, \varepsilon}{\partial \, t} \; + \; \frac{1}{M} \, \frac{\partial \, p}{\partial \, t} \;\; = \;\; \frac{k}{\mu} \, \nabla^2 \, p \; , \tag{2.148}$$

which is the fluid diffusivity equation for the pore fluid pressure in terms of the volumetric strain.

The diffusivity equation for the pore fluid pressure can be written alternatively by using the mean normal stress ($\sigma \; \equiv \; \sigma_{kk}/3$) instead of the volumetric strain (ε_{kk}). For this purpose, the volumetric strain (ε_{kk}) is expressed in terms of the mean normal stress (σ) according to the constitutive equations (2.36), and inserted into eq. (2.148). Then the fluid diffusivity equation for the pore fluid pressure in terms of the mean stress reads:

$$\frac{\alpha}{K} \, \frac{\partial \, \sigma}{\partial \, t} \; + \; \frac{1}{M} \, \frac{K_u}{K} \, \frac{\partial \, p}{\partial \, t} \;\; = \;\; \frac{k}{\mu} \, \nabla^2 \, p \; . \tag{2.149}$$

Both diffusivity equations for the pore fluid pressure (2.148), (2.149) are coupled equations, in the sense that the pore fluid pressure diffusion is coupled with the rate of change either of the volumetric strain (2.148) or of the mean normal stress (2.149). (The terms containing time derivatives of the mean normal stress or of the volumetric strain are mathematically equivalent to pore fluid sources.) Hence, the fluid diffusivity equation cannot be solved independently of the force balance equations. However, there are some particular cases, in which the pore fluid diffusivity equation for the pore fluid pressure and the force balance equations become uncoupled:

- Steady-state conditions, when ($\partial \sigma / \partial \, t \; = \; 0$) or ($\partial \, \varepsilon / \partial \, t \; = \; 0$) holds, and the diffusivity equation becomes the Poisson's equation.
- Constant vertical stress and uniaxial strain.
- Very compressible pore fluid.
- Irrotational displacement field in the infinite domain and zero body forces.

Fluid diffusivity Equation for the Variation of Fluid Content. A homogeneous diffusivity equation uncoupled from the stress/strain can be obtained, provided the variation of fluid content (ζ) is chosen as the pore fluid variable in the diffusivity equation instead of the pore fluid pressure (p). It can be derived from eq. (2.147), which was obtained by combining the fluid continuity equation and Darcy's law, by substituting for the pore fluid pressure (p) according to the constitutive equations and the force balance equations. The Laplacian of the pore fluid pressure in eq. (2.147)

can be expressed in terms of the variation of fluid content (ζ) and the body forces ($\vec{f}$), according to the constitutive equations (2.35) and eq. (2.125). After some algebra, eq. (2.147) leads to the uncoupled fluid diffusivity equation for the variation of fluid content,

$$\frac{\partial \zeta}{\partial t} \;=\; \frac{k}{\mu\, S_a}\, \nabla^2 \zeta \;+\; \frac{k}{\mu}\, \frac{\eta}{G\, S_a}\, \vec{\nabla} \cdot \vec{f}, \qquad\qquad (2.150)$$

where some common relations between the poroelastic parameters were used. In the case of zero body forces ($\vec{f} = \vec{0}$), (2.150) becomes a homogeneous equation. If the boundary conditions for ζ are given, eq. (2.150) can be solved independently of the force balance equations (2.117). Then the solution of the diffusivity equation is used to obtain the solution for displacements.

2.5.3 Governing Equations of Linear Thermoporoelasticity

In the theory of linear thermoporoelasticity, a general problem involves twelve unknowns, which can be determined by solving twelve governing equations. The following physical laws must be considered in order to derive the governing equations:

- Constitutive equations (consisting of seven equations for the bulk material).
- Force balance equations (consisting of three equations for the bulk material).
- Fluid continuity equation (consisting of one equation for the pore fluid).
- Darcy's law (consisting of three equations for the pore fluid).
- Fourier's law (consisting of three equations for the bulk material).
- First law of thermodynamics (consisting of one equation for the bulk material).
- Second law of thermodynamics (consisting of one equation for the bulk material).

Fourier's law is combined with both the first and the second principles of thermodynamics, leading to the single *thermal diffusivity equation*. Hence, a linear thermoelastic problem is concerned with twelve scalar governing equations. In comparison with the isothermal linear poroelasticity, there is one additional governing equation in linear thermoporoelasticity. The set of eleven governing equations is composed of

- seven constitutive equations,
- three force balance equations,
- one fluid diffusivity equation,

- one thermal diffusivity equation.

Fluid Diffusivity Equation. In the case of themoporoelasticity, the *fluid diffusivity equation* can be obtained by taking the same initial steps as in the non-isothermal case, i.e., by combining the fluid continuity equation (2.134) with Darcy's law (2.126), but now accounting also for temperature-driven changes (besides the changes induced by the pore fluid pressure) in the pore fluid density (ρ_f).

In what follows, the derivation of the fluid diffusivity equation of thermoporoelasticity in its general non-linear form is performed. In order to make the derivation as simple as possible, we will confine ourselves to the one-dimensional case, taking into account eq. (2.137). The fluid continuity equation (2.134) in its one-dimensional form reads:

$$\frac{\partial}{\partial t}\left(\rho_f\, n\right)\; +\; \vec{\nabla}\left[\rho_f\, q_x\right]\;=\;0\,. \tag{2.151}$$

After some mathematical manipulations one obtains

$$\rho_f^0\,\frac{\partial n}{\partial t}\; +\; n_0\,\frac{\partial \rho_f}{\partial t}\;=\;-\,\rho_f^0\,\frac{\partial q_x}{\partial x}\; -\; q_x\,\frac{\partial \rho_f}{\partial x}\,, \tag{2.152}$$

and next, dividing the above equation by ρ_f^0 gives

$$\frac{\partial n}{\partial t}\; +\; n_0\,\frac{1}{\rho_f^0}\,\frac{\partial \rho_f}{\partial t}\; +\; \frac{\partial q_x}{\partial x}\; +\; q_x\,\frac{1}{\rho_f^0}\,\frac{\partial \rho_f}{\partial x}\;=\;0\,. \tag{2.153}$$

The following relations for time- and space-differentiations of the pore fluid density (ρ_f) are assumed to be valid, following from the fact that the only variations that can induce changes in the pore fluid density are the variations in the pore fluid pressure (p) and in the temperature (T), i.e., $\rho_f = \rho_f(p, T)$:

$$\frac{\partial \rho_f}{\partial t}\;=\;\frac{\partial \rho_f}{\partial p}\,\frac{\partial p}{\partial t}\; +\; \frac{\partial \rho_f}{\partial T}\,\frac{\partial T}{\partial t}\,, \tag{2.154}$$

$$\frac{\partial \rho_f}{\partial x}\;=\;\frac{\partial \rho_f}{\partial p}\,\frac{\partial p}{\partial t}\; +\; \frac{\partial \rho_f}{\partial T}\,\frac{\partial T}{\partial t}\,. \tag{2.155}$$

By making use of the above two relations in eq. (2.153), we obtain

$$\frac{\partial n}{\partial t}\; +\; n_0\left(\frac{1}{\rho_f^0}\,\frac{\partial \rho_f}{\partial p}\,\frac{\partial p}{\partial t}\; +\; \frac{1}{\rho_f^0}\,\frac{\partial \rho_f}{\partial T}\,\frac{\partial T}{\partial t}\right)\; +\; \frac{\partial q_x}{\partial x}\; +$$
$$+\; q_x\left(\frac{1}{\rho_f^0}\,\frac{\partial \rho_f}{\partial p}\,\frac{\partial p}{\partial x}\; +\; \frac{1}{\rho_f^0}\,\frac{\partial \rho_f}{\partial T}\,\frac{\partial T}{\partial x}\right)\;=\;0\,. \tag{2.156}$$

A substitution for q_x in the above equation according to Darcy's law (2.126) leads to

$$\frac{\partial n}{\partial t} + n_0 \left(\frac{1}{\rho_f^0} \frac{\partial \rho_f}{\partial p} \frac{\partial p}{\partial t} + \frac{1}{\rho_f^0} \frac{\partial \rho_f}{\partial T} \frac{\partial T}{\partial t} \right) - \frac{\partial}{\partial x} \left[\frac{k}{\mu} \frac{\partial p}{\partial x} \right] -$$
$$- \frac{k}{\mu} \left(\frac{1}{\rho_f^0} \frac{\partial \rho_f}{\partial p} \left(\frac{\partial p}{\partial x} \right)^2 + \frac{1}{\rho_f^0} \frac{\partial \rho_f}{\partial T} \frac{\partial T}{\partial x} \frac{\partial p}{\partial x} \right) = 0 . \tag{2.157}$$

Next, recognizing the pore fluid bulk compressibility (β_f) definition (2.72), and the pore fluid thermal expansion coefficient (α_f) definition (2.104) in eq. (2.157) leads to

$$\frac{\partial n}{\partial t} + n_0 \left(\beta_f \frac{\partial p}{\partial t} - \alpha_f \frac{\partial T}{\partial t} \right) - \frac{\partial}{\partial x} \left[\frac{k}{\mu} \frac{\partial p}{\partial x} \right] -$$
$$- \frac{k}{\mu} \left(\beta_f \left(\frac{\partial p}{\partial x} \right)^2 - \alpha_f \frac{\partial T}{\partial x} \frac{\partial p}{\partial x} \right) = 0 . \tag{2.158}$$

The generalization of eq. (2.158) to the 3-D case is straightforward:

$$\frac{\partial n}{\partial t} + n_0 \left(\beta_f \frac{\partial p}{\partial t} - \alpha_f \frac{\partial T}{\partial t} \right) - \vec{\nabla} \left[\frac{k}{\mu} \vec{\nabla} p \right] -$$
$$- \frac{k}{\mu} \left(\beta_f \left(\vec{\nabla} p \right)^2 - \alpha_f \vec{\nabla} T \cdot \vec{\nabla} p \right) = 0 . \tag{2.159}$$

Hereafter, we will refer to the above equation as the *general non-linear fluid diffusivity equation* of thermoporoelasticity.

Now, the non-linear equation (2.158) will be used to derive the fluid diffusivity equation of linear thermoporoelasticity. For this purpose, let us perform some mathematical manipulations on (2.158) and rewrite it in the form:

$$\frac{\partial n}{\partial t} + n_0 \beta_f \frac{\partial p}{\partial t} - n_0 \alpha_f \frac{\partial T}{\partial t} - \frac{k}{\mu} \frac{\partial^2 p}{\partial x^2} - \frac{\partial}{\partial x} \left(\frac{k}{\mu} \right) \frac{\partial p}{\partial x} -$$
$$- \frac{k}{\mu} \beta_f \left(\frac{\partial p}{\partial x} \right)^2 + \frac{k}{\mu} \alpha_f \frac{\partial T}{\partial x} \frac{\partial p}{\partial x} = 0 . \tag{2.160}$$

It is clear that the three last terms on the l.h.s. are non-linear, hence can be omitted within a linear framework. Thus the fluid diffusivity equation of linear thermoporoelasticity reads:

$$\frac{\partial n}{\partial t} + n_0 \beta_f \frac{\partial p}{\partial t} - n_0 \alpha_f \frac{\partial T}{\partial t} - \frac{k}{\mu} \frac{\partial^2 p}{\partial x^2} = 0 . \tag{2.161}$$

Or, written in the 3-D form:

$$\frac{\partial n}{\partial t} + n_0 \beta_f \frac{\partial p}{\partial t} - n_0 \alpha_f \frac{\partial T}{\partial t} - \frac{k}{\mu} \vec{\nabla}^2 p = 0 . \tag{2.162}$$

Fourier's Law. Fourier's law, called also *heat diffusion law*, is an empirically derived law of heat conduction which states that the rate by which the heat (Q) is transfered per unit surface of a homogeneous medium (i.e., the heat flux) is proportional to the temperature gradient. The differential form of Fourier's law reads

$$\vec{q}_{th} \;=\; -\,k_{th}\,\vec{\nabla}\,T\,, \tag{2.163}$$

where $\vec{q}_{th}$ is the heat flux.

The one-dimensional form of Fourier's law is often used:

$$q_{th} \;=\; -\,k_{th}\,\frac{\partial\,T}{\partial\,x}\,, \tag{2.164}$$

where q_{th} is the heat flux in the x-direction.

First Law of Thermodynamics. The first law of thermodynamics is an expression of the energy conservation principle for a thermodynamic system. It states that energy cannot be created or destroyed, it can be only transformed from one form to another. Applied to the case of a fluid-saturated porous material, the first law of thermodynamics reads:

$$\dot{U} \;+\; \dot{K} \;=\; \dot{W}_e \;+\; \dot{Q}\,. \tag{2.165}$$

Here, $\dot{U}$, $\dot{K}$, $\dot{W}_e$, and $\dot{Q}$ are the rates of, respectively, internal energy, kinetic energy, work of external forces, and heat supply to the system. After some mathematical manipulations, and by neglecting the rate of kinetic energy ($\dot{K} \;=\; 0$), one obtains (see Charlez (1991) for details of the derivation)

$$\dot{u}_v \;+\; \vec{\nabla}\,\cdot\,(\,h_m\,\rho_f\,\vec{q}\,) \;=\; \sigma_{ij}\,\dot{\varepsilon}_{ij} \;-\; \vec{\nabla}\,\cdot\,\vec{q}_{th}\,. \tag{2.166}$$

Here, u_v denotes the *bulk internal energy* per unit volume, and h_m denotes the *specific pore fluid enthalpy*. The latter can be expressed as

$$h_m \;=\; u_m \;+\; \frac{p}{\rho_f}\,, \tag{2.167}$$

where u_m denotes the *specific pore fluid internal energy*.

Second Law of Thermodynamics. The second law of thermodynamics, introduced by eq. (2.33), can be expressed by various formulations. One of them, often used in continuum mechanics, is the *Clausius-Duhem inequality*. This inequality states that the dissipation of energy is never negative. In the case of reversible processes, the Clausius-Duhem inequality becomes equality, expressing that there is no dissipation of energy. In order to derive the Clausius-Duhem formulation of the second law of thermodynamics, let us write eq. (2.33) in the following integral form (in the Lagrangian formulation):

$$\frac{\partial}{\partial\,t}\,\int_{V_b^0}\,s_0\,dV_b^0 \;+\; \int_{A_0}\,s_0^f\,\vec{q}\,\cdot\,\vec{n}_0\,dA_0 \;\geq\; -\,\int_{A_0}\,\frac{\vec{q}_{th}\,\cdot\,\vec{n}_0}{T}\,dA_0\,. \tag{2.168}$$

Here, s_0 is the entropy per unit bulk volume, s_0^f is the entropy of the pore fluid per unit (pore fluid) volume, A_0 is the surface area of the bulk volume, and $\vec{n}_0$ is the normal vector oriented to the exterior. The local form of eq. (2.168) is obtained by applying the divergence theorem,

$$T\,\dot{s}_0 \;+\; T\,\vec{\nabla}\cdot\left(s_0^f\,\vec{q}\right) \;\geq\; -\,\vec{\nabla}\cdot\vec{q}_{th} \;+\; \frac{\vec{q}_{th}}{T}\cdot\vec{\nabla}\,T\,. \tag{2.169}$$

Using the first law of thermodynamics (2.165) in the above equation, performing some mathematical manipulations, and introducing the *volumetric free energy* ψ_0, and the *specific free pore fluid enthalpy* g_m, respectively as

$$\psi_0 \;=\; u_v^0 \;-\; T\,s_0\,, \tag{2.170}$$

$$g_m \;=\; h_m \;-\; T\,s_m\,; \quad s_m\,\rho_f \;=\; s_0^f\,, \tag{2.171}$$

the Clausius-Duhem inequality is obtained in the following form:

$$\Phi \;=\; \Phi_1 \;+\; \Phi_2 \;\geq\; 0\,. \tag{2.172}$$

Here, the *dissipation of energy* (Φ) is given by a superposition of the *intrinsic dissipation* Φ_1, defined as

$$\Phi_1 \;=\; \sigma_{ij}\cdot\dot{\varepsilon}_{ij} \;-\; s_0\,\dot{T} \;+\; g_m\,\dot{m}_f \;-\; \dot{\psi}_0 \;\geq\; 0\,, \tag{2.173}$$

and the *thermohydraulic dissipation* Φ_2, defined as

$$\Phi_2 \;=\; -\,\frac{\vec{q}_{th}}{T}\cdot\left(\vec{\nabla}\,T\right) \;-\; \rho_f\,\vec{q}\cdot\left(\vec{\nabla}\,g_m \;+\; s_m\,\vec{\nabla}\,T\right) \;\geq\; 0\,. \tag{2.174}$$

Under conditions of reversible processes, which is the case of thermoporoelasticity, the Clausius-Duhem inequality becomes an equality, i.e., the dissipation is zero ($\Phi = 0$). Following eqs. (2.172), (2.173), and (2.174), the Clausius-Duhem equality for a thermoporoelastic system reads:

$$\sigma_{ij}\cdot\dot{\varepsilon}_{ij} \;-\; s_0\,\dot{T} \;+\; g_m\,\dot{m}_f \;-\; \dot{\psi}_0 \;-\; \frac{\vec{q}_{th}}{T}\cdot\left(\vec{\nabla}\,T\right) \;-$$
$$-\;\rho_f\,\vec{q}\cdot\left(\vec{\nabla}\,g_m \;+\; s_m\,\vec{\nabla}\,T\right) \;=\; 0. \tag{2.175}$$

Thermal Diffusivity Equation. By combining the Clausius-Duhem equality (2.175) with Fourier's law (2.163), the thermal diffusivity equation of thermoporoelasticity can be derived. Its one-dimensional form reads (Mase and Smith, 1985):

$$\frac{\partial}{\partial x} \left(k_{th} \frac{\partial T}{\partial x} \right) - \rho_f c_f q_x \left(\frac{\partial T}{\partial x} \right) + n \alpha_f \left(\frac{\partial p}{\partial t} + \frac{q_x}{n} \frac{\partial p}{\partial x} \right) T +$$
$$+ Q_s = \rho_b c_b \frac{\partial T}{\partial t} . \tag{2.176}$$

Here, Q_s denotes the *heat source*, and ρ_b is the density of the bulk material. Terms in the above equation represent in order from left to right the rate of heat conduction, the rate of heat convection, the rate of heating due to the work done on the pore fluid, the rate of heating due to the heat source (Q_s), and the rate of internal heat storage (Mase and Smith, 1985). The local thermal equilibrium between the solid and fluid phases of the bulk material is implicitly assumed in the equation. A further modification of eq. (2.176) consists in substituting for q_x according to Darcy's law (2.126), and using eq. (2.113):

$$\frac{\partial}{\partial x} \left(k_{th} \frac{\partial T}{\partial x} \right) + \rho_f c_f \frac{k}{\mu} \left(\frac{\partial p}{\partial x} \right) \left(\frac{\partial T}{\partial x} \right) + n \alpha_f T \frac{\partial p}{\partial t} -$$
$$- \alpha_f \frac{k}{\mu} T \left(\frac{\partial p}{\partial x} \right)^2 + Q_s = c_e \frac{\partial T}{\partial t} . \tag{2.177}$$

The thermal diffusivity equation derived so far is a non-linear equation. However, within the framework of linear thermoporoelasticity, only the linearized form of the equation is treated. The thermal diffusivity equation of linear thermoporoelasticity is obtained from eq. (2.176) by neglecting the non-linear convection term, which is the second one on the l.h.s.,

$$k_{th} \frac{\partial^2 T}{\partial x^2} + n \alpha_f T \frac{\partial p}{\partial t} - \alpha_f \frac{k}{\mu} T \left(\frac{\partial p}{\partial x} \right)^2 + Q_s = c_e \frac{\partial T}{\partial t} , \tag{2.178}$$

where the assumption of a spatially constant thermal conductivity (k_{th}) was applied. The 3-D form of the *thermal diffusivity equation of linear thermoporoelasticity* is obtained straightforwardly from eq. (2.178):

$$k_{th} \vec{\nabla}^2 T + n \alpha_f T \frac{\partial p}{\partial t} - \alpha_f \frac{k}{\mu} T \left(\frac{\partial p}{\partial x} \right)^2 + Q_s = c_e \frac{\partial T}{\partial t} . \tag{2.179}$$

3 Thermal Pressurization of Pore Fluid Process During Earthquake Slip

3.1 Overview

At the present level of knowledge, there are four possible candidate mechanisms that may play a role in the dynamic fault weakening process during an earthquake. However, convictive arguments exist only for the two of them: the thermal pressurization of pore fluid and the flash heating of microscopic asperity contacts. Today, they are assumed to be the primary fault weakening mechanisms acting in combination (at least) during large shallow earthquakes occurring on mature faults. Strong evidence is lacking to support the other two mechanisms - the melting and the silica gel formation, and further research is needed to decide whether they play a significant role during earthquakes or not.

In what follows, an analysis of the thermal pressurization of pore fluid process is presented. It is based on the theory of thermoporoelasticity introduced in the previous chapter, and on the latest results on the topic - both the theoretical and experimental. Especially the studies of Mase and Smith (1985), Bizzarri and Cocco (2006a), Rempel and Rice (2006), and Rice (2006) are followed. The goal of the analysis is to find the most appropriate, but as simple as possible physical model of the thermal pressurization process, with proper governing equations and adequate values of relevant parameters. The investigation starts with a presentation of geological, laboratory and theoretical findings, resulting in several assumptions and constraints on the physical model and in adequate values of relevant model parameters. Then the physical model is proposed, starting with the model geometry and material properties, followed by a description of the physical mechanisms occurring under given conditions, and ending with the governing equations of the model. One of the proposed governing equations is nonlinear. It is the most significant difference between the physical model proposed here and the commonly used models with linear equations (Bizzarri and Cocco, 2006a), (Rempel and Rice, 2006), (Rice, 2006). After the linearization process, the governing equations presented here become identical to the commonly used governing equations.

However, the non-dimensionalisation of the derived non-linear governing equations and quantification of the individual terms shows that some of the non-linear terms cannot be neglected - at least with the present level of uncertainty in values of various model parameters.

3.2 Physical Model

Following the theory of thermoporoelasticity introduced in the previous chapter, and the latest geological, laboratory, and theoretical results, we will try to propose a physical model of the thermal pressurization of pore fluid process during earthquake slip.

3.2.1 Geological, Laboratory, and Theoretical Findings

The physical model must follow the latest findings in the field. The findings related to the thermal pressurization of pore fluid process can be divided into three groups according to their origin:

- Geological.
- Laboratory.
- Theoretical.

Geological Findings. Recently started (2004) drilling project in the San Andreas fault system in California (called SAFOD - San Andreas Fault Observatory at Depth) has considerably contributed to our knowledge of the structure of active major fault zones, as it is for the first time when scientists drill into a seismically active section of a fault (at a depth approximately $3000\ m$ below the Earth's surface), and install there instruments (seismometers, strainmeters, fluid and temperature gauges). Observational results of the SAFOD project related to the structure of the fault zone are in a good agreement with those accumulated by previous studies of: the exhumed inactive North Branch San Gabriel fault and the Punchbowl fault of the San Andreas system, the Median Tectonic Line fault system in Japan, the Hanaore fault in southwest Japan, and the Nojima fault in Japan (Rice, 2006).

The most important fact resulting from the in-situ observations is the extreme thinness of shear zones, i.e., those parts of fault zones, where the slipping takes place during

an earthquake. They are commonly about $1\ mm$ (or less) thick, which is much lesser than had been previously believed. Hereafter, the shear zone will be called *principal slipping zone* (PSZ) (Sibson, 2003). Moreover, within the PSZ, most of the shearing seems to have been accommodated within a zone of apparent thickness of only $100 - 300\ \mu m$ (Rice, 2006). The PSZ is therefore sometimes called *principal slipping surface*. The PSZ is illustrated in Fig. 2.
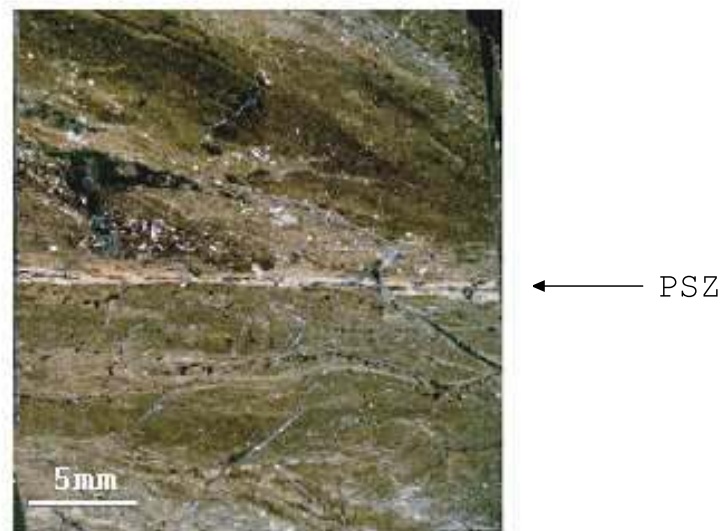

Figure 2. The principal slipping zone of the Punchbowl fault, based on Chester et al. (2005).

While the slip is localized to an extremely thin zone, the net zone in which the host rock is deformed, called *damage zone*, is relatively wide - its thickness ranges from $10s\ m$ to $100s\ m$. The damage zone is highly fractured and anisotropic (Rice and Cocco, 2006). The PSZ is embedded in the finely granulated, "ultracataclastic" (i.e., composed of an extremely fine-grain product of frictional wear) *fault core* with thickness ranging from approximately $10s\ mm$ to $10s\ cm$. The fault core lies within the *gouge zone*, having thickness of about $1 - 10s\ m$, composed of a "cataclastic" material which is less finely granulated than that of the core, possibly with a foliated structure. A representative model of major fault zones, resulting from geological observations is given in Fig. 3.

Laboratory Findings. The physical model of the thermal pressurization of pore fluid process is constrained both with laboratory determined properties of fault core mate-
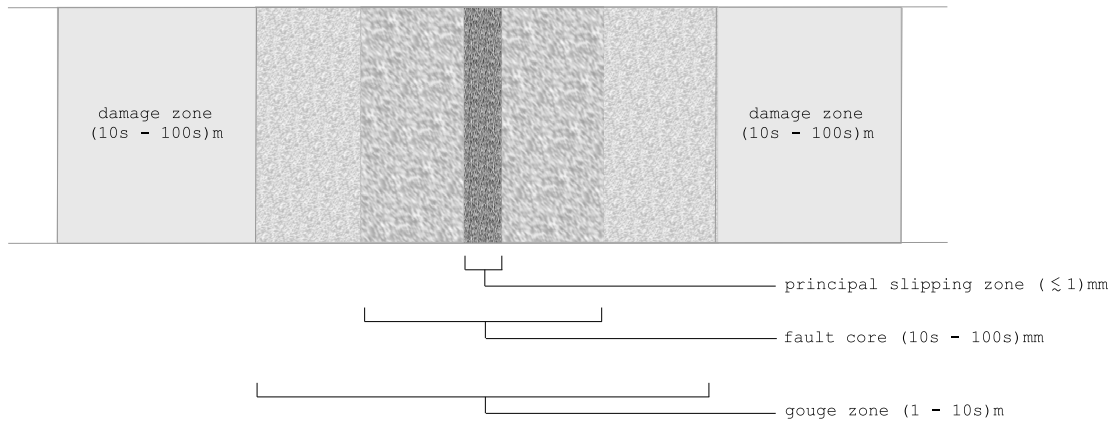
Figure 3. A schematic fault zone model. (It is not to scale.)

rials, and with results of laboratory friction experiments performed at high slip rates. Thanks to the SAFOD drilling project, actual rock samples acquired at hypocentral depths of an active fault can be now investigated in the laboratory, and friction experiments can be performed with them. Moreover, almost the coseismic slip rates and coseismic confining stresses have been recently achieved in the friction experiments (Rempel, 2006), see the works of Tsutsumi and Shimamoto (1997), Di Toro et al. (2004, 2006), Prakash (2004), Prakash and Yuan (2004), Mizoguchi and Shimamoto (2004).

Laboratory friction experiments performed with fault materials both from the Nojima fault (of the 1995 Kobe Earthquake in Japan) (Lockner et al., 2000) and from the San Andreas fault ("phase 1" and "phase 2" of the SAFOD project) (Morrow et al., 2007) suggest that the strength of fault zones increases gradually with the distance from the shear axis, having the lowest friction coefficient within the fault core ($f \approx 0.55$ (Nojima fault), $f \approx 0.4$ (San Andreas fault)), and the highest friction coefficient in the damage zone ($f \approx 0.8$ (Nojima fault), $f \approx 0.5$ (San Andreas fault)). The lower strength of the fault core is probably related to the its high clay content (relative to the surroundings). Furthermore, in both experiments, the measured coefficients of friction were greater for materials acquired from greater depths than for those from deeper fault regions. For comparison, Tembe et al. (2006) have measured coefficient of friction value of approximately 0.45 within the fault core of the San Andreas fault, and 0.6 in the surroundings. The experiments imply that:

- Faults are actually weaker than predicted by Byerlee's law (i.e., by the result of low speed friction experiments performed at slip rates far lower than the coseismic ones, universally revealing friction coefficients between 0.6 and 0.9 for almost all rock types), with the fault core friction coefficients of only about 0.45.

- However, even the values of about 0.45 of the friction coefficient are not sufficient to explain the so-called heat flow paradox, i.e., the lack of measured heat outflow near major faults due to the frictional sliding, theoretically predicted according to the measured values of the shear zone thickness, friction coefficient, and slip rate. It suggests that some dynamic weakening mechanisms should occur during an earthquake in order to rapidly reduce the fault strength once the slip begins, thus reducing the frictional heat outflow and limiting the temperature rise. Much lower values of the friction coefficient would be theoretically needed to explain the heat flow paradox without considering any dynamic weakening mechanisms (of about 0.2 or less for the San Andreas fault (Tembe et al., 2006)). Hence, the hypothesis of *statically strong but dynamically weak faults* may be possibly valid (Lapusta and Rice, 2003), (Noda et al., 2008), (Sone and Shimamoto, 2008).

The laboratory measurements of the fault rock samples are now aimed to put better constraints on material parameters such as the permeability (k), the porosity (ϕ), various compressibilities and other (thermo)poroelastic parameters. Especially the permeability is a crucial parameter in the physical model of the thermal pressurization process, since its value determines whether this process could be effective during an earthquake or not. If the permeability value of the fault core rock was too high, the pore fluid would be drained away rather than being pressurized. The common result of the measurements performed by Lockner et al. (2000), Mizoguchi et al. (2000), Wibberley and Shimamoto (2003), Sulem et al. (2004), and Noda and Shimamoto (2005) is that the fault core has a much lower permeability than the surroundings. The permeability within the fault core can be even three orders of magnitude lower than that in the damage zone, with values ranging from about $10^{-19} \ m^2$ in the fault core, to $10^{-16} \ m^2$ in the damage zone (Rice, 2006). Furthermore, various measurements imply that permeability values considerably vary from fault to fault, and also along the same

fault. Thus the permeability is often designated as the most uncertain parameter of fault zone materials (Wibberley, 2002).

Further results of laboratory measurements are presented in Tab. 2.

Theoretical Findings and Assumptions. There is a great amount of theoretical considerations regarding the thermal pressurization of pore fluid process. Therefore, we propose only the resulting assumptions and constraints which we adopt in the physical model. The used assumptions and constraints are, of course, in agreement with the already discussed geological and laboratory results. Some of the assumptions do not arise from the theory neither as the most appropriate nor the only possible model properties, but unless they are evincibly inadequate for the model, we adopt them in order to make the physical model as simple as possible.

Thus the following assumptions and constraints are used in the physical model of the thermal pressurization process:

- The problem is treated in one dimension in the fault-normal direction. The fault-normal direction coincides with the x-coordinate axis. Thus the only component of the total stress (σ_{ij}) entering the problem is σ_{xx} in the fault-normal direction, hereafter denoted as σ_n.

  The length scales over which the key quantities, such as the slip rate (ΔV_s), or the shear stress (τ) vary are commonly far larger than the length scale in the fault normal direction which is affected by the thermal pressurization of pore fluid process (i.e., by the diffusion of heat and pore fluid).

- The fault-normal stress (σ_n) is assumed to remain constant during the earthquake slip (Bizzarri and Cocco, 2006a), (Rice, 2006).

- Body forces are neglected ($\vec{f} = 0$).

  The only body force which could possibly affect the process is the gravitational force. However, its effect seems to be negligible over the short time interval of earthquake slip (Delaney, 1982), (Mase and Smith, 1985).

- The slip rate (ΔV_s) is assumed to be constant (both spatially and temporally).

- The solid phase of the rock is assumed to be effectively incompressible, i.e., $\beta_s = 0$. Consequently, Biot-Willis coefficient (α) equals one according to eq. (2.69):

$$\alpha \;=\; 1 \;-\; \frac{\beta_s}{\beta_b} \;=\; 1 \,. \tag{3.1}$$

- The thermal pressurization process is assumed to act only within the fault core. Theoretical estimations of characteristic thermal diffusion and pore fluid diffusion length scales, i.e., the typical fault-normal distances measured from the fault plane over which the temperature and pore-fluid pressure changes are significant during the slip duration, suggest values from about few millimeters to few centimeters (Rice, 2006). The thickness of these thermal and hydraulic boundary layers is comparable to the typical fault core thickness (Rempel and Rice, 2006). Behind those distances, the changes in the pore fluid pressure and temperature are negligible, thus the thermal pressurization becomes ineffective. Hence, we use $1\,cm$ as the characteristic length scale for the thermal pressurization process.

- The material parameters are assumed to be spatially homogeneous and time-independent. The validity of the assumption of homogeneity is questionable at least due to the large spatial variations in the value of permeability (k) in the fault-normal direction. However, the assumption could be admissible, as the thermal pressurization process effectively occurs at very small fault-normal length scales (where the permeability variations are possibly negligible). The assumption of time-independence is adopted, although there is some evidence on significant time variations of various parameters during an earthquake slip, e.g., the porosity (n), or the drained bulk compressibility (β_b) (Wibberley and Shimamoto, 2005).

- The pore fluid pressure changes induced by the propagating rupture front are not taken into account (only the slip-induced pore fluid pressure changes are considered).
  The pore-fluid pressure changes induced by the propagating rupture front are assumed to be negligible in comparison with the pore-fluid pressure changes induced by the frictional sliding (Mase and Smith, 1985).

- Darcy's law in the linear form given by eq. (2.126) is assumed to be applicable. First, this assumption is valid under conditions of sufficiently small Reynolds number. Calculating the Reynolds number for the thermal pressurization problem according to eq. (2.130) gives the value of the order of 0.001, which suggests that we are concerned with laminar flows, hence Darcy's law is applicable.

Second, Darcy's law is an empirical law, derived on the basis of experiments performed at constant, standard pressures and temperatures (i.e., at about 101.325 kPa and 293.15 K) with water. Under such conditions, the water is almost incompressible (its average compressibility under standard conditions is about one order smaller than during the thermal pressurization process), and no thermal expansion occurs (the conditions are isothermal). There is a hypothesis suggesting that the validity of Darcy's law in the linear form given by eq. (2.126) is conditioned by the pore fluid incompressibility ($\beta_f = 0$). The hypothesis follows from theoretical studies (e.g., Wang, L. (2000)), but also from theoretical attempts at Darcy's law derivation from basic principles (Navier-Stokes equations) (e.g., Whitaker (1986), Bear (1988)). Nevertheless, experiments suggest that also the supercritical water flows through a porous material according to the linear form of Darcy's law (Fauvel et al., 2004). Therefore, we adopt the simplest linear form of Darcy's law in this thesis and assume its approximate validity, despite the fact that we are dealing with a compressible pore fluid in the thermal pressurization process.

3.2.2 Values of Model Parameters

According to the results of geological, laboratory and theoretical investigations, we propose a set of relevant parameters values for the physical model of the thermal pressurization of pore fluid process in Tabs. 1, 2 and 3. The values are often in accordance with Rice (2006). Some of them, however, are different, especially those for various material parameters which are strongly dependent on the pore fluid pressure (p) and/or on the temperature (T). We have tried to find the most appropriate values for those parameters, under the pressure-temperature conditions characteristic for the thermal pressurization process in the typical seismogenic depth (7 km), i.e., $p \approx 100\ MPa$, $T \approx 1000\ K$ (Rice, 2006), see Tab. 3. The characteristic pressure-temperature values and the values of material parameters which vary considerably during the earthquake slip (e.g., the pore fluid density) were computed as the time-averaged values during the slip at a point, i.e., 10 s.

Table 1. Values of model parameters. A: Geological, seismological and observational parameters.

Parameter	Symbol	Value	Unit
PSZ half-width	w_s	1	mm
Fault core half-width	w_c	5	cm
Slip	ΔU	10	m
Slip rate	ΔV_s	1	$m.s^{-1}$
Slip duration at a point	t_{tot}	10	s

Table 2. Values of model parameters. B: Laboratory determined parameters.

Parameter	Symbol	Value	Unit
Permeability [1]	k	10^{-19}	m^2
Porosity [1]	n	0.04	1
Density of the bulk material	ρ_b	2700	$kg\ m^{-3}$
Drained bulk compressibility [2]	β_b	7×10^{-11}	Pa^{-1}
Effective volumetric heat capacity [1]	c_e	2.7×10^6	$Pa\ K^{-1}$
Coefficient of friction [3]	f	0.45	1

[1] According to Rice (2006).

[2] According to Wibberley and Shimamoto (2003).

[3] According to Tembe et al. (2006).

3.2.3 Geometry and Material Properties

Geometry. The geometry of the model is relatively simple as a consequence of the one-dimensional treatment of the process, and due to another assumptions discussed up to now. The following fault zone structure is considered: the *fault plane* $(x = 0)$ is embedded in the finite-thickness principal slipping zone (PSZ) $(-w_s \leq x \leq +w_s)$. The PSZ lies within the fault core $(-w_c \leq x \leq +w_c)$ which is further surrounded by the damage zone. The geometry of the model is illustrated in Fig. 4, and the relevant values are given in Tab. 1.

Material Properties. The material is considered to be water-saturated, linearly thermoporoelastic, macroscopically homogeneous and isotropic. Moreover, the values of material parameters are assumed to be constant in time. The values of relevant material parameters are given in Tabs. 2 and 3.

Table 3. Values of model parameters. C: Theoretically determined parameters.

Parameter	Symbol	Value	Unit
Fault-normal stress (absolute value) [1]	σ_n	196	MPa
Pore fluid pressure in the initial state [2]	p_0	70	MPa
Pore fluid pressure	p	133	MPa
Terzaghi's effective pressure	P_e'	63	MPa
Temperature in the initial state [3]	T_0	480	K
Temperature	T	1000	K
Pore fluid compressibility [4]	β_f	4.5×10^{-9}	Pa^{-1}
Pore fluid density [5]	ρ_f	340	$kg\ m^{-3}$
Pore fluid dynamic viscosity [6]	μ	5.7×10^{-5}	$Pa\ s$
Pore fluid specific heat capacity [7]	c_f	2.3×10^3	$J\ kg^{-1}\ K^{-1}$
Pore fluid thermal expansivity [8]	α_f	2.5×10^{-3}	K^{-1}
Solid thermal expansivity [9]	α_s	2.4×10^{-5}	K^{-1}
Hydraulic diffusivity [10]	α_{hy}	10^{-5}	$m^2\ s^{-1}$
Thermal diffusivity [11]	α_{th}	0.5×10^{-6}	$m^2\ s^{-1}$
Effective thermal conductivity of the bulk material [12]	k_{th}	1.35	$J\ m^{-1}\ s^{-1}\ K^{-1}$

[1]Calculated as the lithostatic stress at the typical seismogenic depth (7 km).

[2]Calculated as the hydrostatic pressure at the typical seismogenic depth (7 km).

[3]Calculated at the typical seismogenic depth (7 km), using the geothermal gradient 30 $K\ km^{-1}$.

[4]According to Rice (2006).

[5]According to Ueda et al. (2002), and NIST WebBook Chemie (2009).

[6]According to NIST WebBook Chemie (2009).

[7]According to NIST WebBook Chemie (2009).

[8]According to Engineeringtoolbox (2009).

[9]According to Rempel (2006).

[10]Calculated according to eq. (2.101).

[11]According to Rice (2006).

[12]Calculated according to eq. (2.114).

l

3.2.4 Physical Mechanism

The physical model of the thermal pressurization of pore fluid process includes the following physical mechanism. After the slip on the fault is initiated ($\Delta V_s \neq 0$), the frictional heat (Q_s) is continuously generated within the PSZ due to the frictional sliding. The generated heat induces a temperature rise both of the pore fluid and of the solid matrix comprising the fault rock. The thermal expansion coefficient of the pore fluid (α_f) is, however, about two orders of magnitude greater than that of the solid
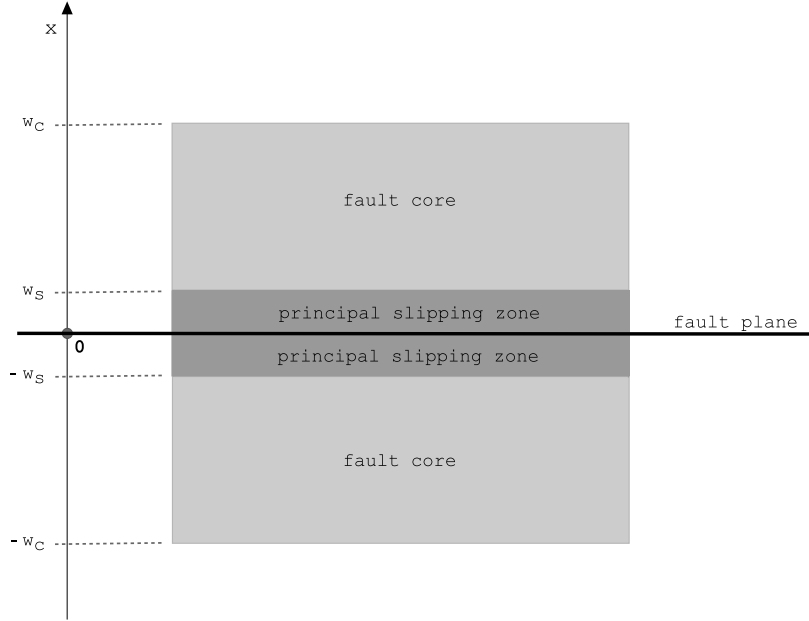
Figure 4. Model geometry. (It is not to scale.)

material (α_s). Hence, the pore fluid tends to increase its volume much more than the solid does, but the increase of the pore fluid volume is suppressed by the surrounding solid matrix. Thus the pore fluid becomes pressurized rather than being expanded. According to eq. (2.41), Terzaghi's effective pressure law for the thermal pressurization process can be written as

$$ P_e^{'} \;=\; -\,\sigma_n \;-\; p\,. \tag{3.2} $$

The increase in the pore fluid pressure (p) will cause a decrease in Terzaghi's effective pressure $(P_e^{'})$, provided the assumption of constant normal stress (σ_n) holds during the earthquake slip. (We recall that the opposite sign conventions hold for the fault-normal stress (σ_n) and for the pore fluid pressure (p): the stress is negative in compression, while the pressure is positive in compression.)

Consequently, following the friction law in the general form,

$$ \tau \;=\; f\,P_e^{'}\,, \tag{3.3} $$

the *fault strength* (τ) will be decreased, and the earthquake slip thus further promoted. The heat diffusion process (i.e., the heat conduction in the fault-normal direction, represented by the heat flux q_{th}, from the heat source (Q_s) localized within the PSZ) is accompanied by the hydraulic diffusion process (i.e., the pore fluid flow with Darcy's

velocity q in the fault-normal direction). The pore fluid flow is induced by the pore fluid pressure gradient in the fault-normal direction which arose from the temperature gradient in the fault-normal direction. For a given time instant, different temperature changes along the x-coordinate axis induce different pore-fluid pressure changes along the x-coordinate axis (both due to the thermal expansion phenomenon and the pore fluid compressibility), thus the pressure gradient is induced in the fault-normal direction. The situation is illustrated in Fig. 5.
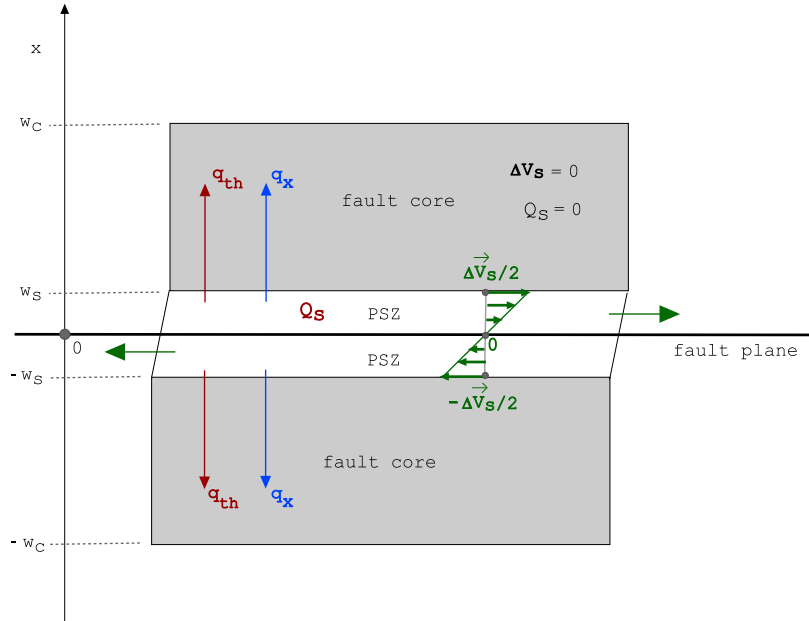

Figure 5. A schematic illustration of the thermal pressurization process mechanism.

Frictional Heating. . The crucial mechanism of the thermal pressurization process is the frictional heating of the fault. In order to incorporate this mechanism into the physical model, an appropriate heat source must be chosen. According to the latest geological findings regarding the extreme thinness of the PSZ, and the theoretical considerations made by Bizzarri and Cocco (2006a), Rice (2006), Rempel and Rice (2006), among others, the heat source can be most appropriately represented either as a finite thickness slipping layer within which the heat is uniformly generated (i.e., the PSZ in our model), or as a planar heat source corresponding to the fault plane. The latter could be a good representation of the heat source for sufficiently long slip distances, but only if the thickness of the PSZ is small compared to the thickness

of hydraulic and thermal boundary layers. This might hold if the PSZ thickness was about $300\,\mu m$ or less. Although it is not impossible that the PSZ is so extremely thin, according to the present knowledge the typical PSZ thickness values are still higher ($\approx\ 1\,mm$). Hence, the model of the heat uniformly generated within the finite-thickness PSZ is adopted here. It corresponds to the situation when the slip rate (ΔV_s) profile across the PSZ is linear, such that the slip rate of the border walls of the PSZ (i.e., the planes located at $x\ =\ -w_s$ and $x\ =\ w_s$) equals ΔV_s. The slip rate outside the PSZ is zero. Then the heat source can be defined as follows (Bizzarri and Cocco, 2006a), (Rempel and Rice, 2006):

$$Q_s\ (\ t\ )\ =\ \frac{\tau\ (\ t\ )\ \Delta V_s}{2\ w_s}\ ;\quad |x|\ \leq\ w_s\ .$$
$$Q_s\ =\ 0\ ;\quad |x|\ >\ w_s\ ,$$
$$(3.4)$$

Here, the assumption that all the frictional work is converted into the heat was used (Cardwell et al., 1978), (Scholz, 2002). The above relation can be rewritten according to the friction law (3.3) as

$$Q_s\ (\ t\ )\ =\ \frac{f\ P_e'\ \Delta V_s}{2\ w_s}\ ;\quad |x|\ \leq\ w_s\ ,$$
$$Q_s\ =\ 0\ ;\quad |x|\ >\ w_s\ ,$$
$$(3.5)$$

which can be further rewritten according to Terzaghi's effective pressure law (3.2) as follows:

$$Q_s\ (\ t\ )\ =\ f\ (\ -\sigma_n\ -\ p\ (\ t\ )\ )\frac{\Delta V_s}{2\ w_s}\ ;\quad |x|\ \leq\ w_s\ ,$$
$$Q_s\ =\ 0\ ;\quad |x|\ >\ w_s\ .$$
$$(3.6)$$

Here, the pore fluid pressure (p) is assumed to be represented by the fault-plane value at every spatial position within the PSZ, i.e., $p\ (t)\ =\ p\ (\ t\ ,\ x\ =\ 0\ )$. In other words, p does not vary with the x-coordinate. Such simplification is often used in the physical models of the thermal pressurization process (Bizzarri and Cocco, 2006a), (Rempel and Rice, 2006). It should not affect the results considerably, since the typical pore fluid pressure gradients at the small distances corresponding to the half-width of the PSZ ($\approx\ 1\,mm$) are relatively small.

Alternatively to the presented heat source model with a uniform heat generation, Andrews (2002) has proposed a model with a Gaussian distribution of the heat production within the PSZ. However, laboratory experiments performed by Mair and Marone

(2000) suggest that the uniform distribution of the heat within the PSZ might be an appropriate heat source representation. Therefore, in agreement with Bizzarri and Cocco (2006a), Cardwell et al. (1978), and Fialko (2004), we use the uniform heat source in the model.

3.2.5 Governing Equations

Since the basic assumptions of thermoporoelasticity are adopted in the physical model (i.e., the solid material is homogeneous and isotropic, and all the pores are interconnected), and we assume that all the processes are reversible, the thermal pressurization of pore fluid process can be categorized as a thermoporoelastic problem. Therefore, following the discussion on thermoporoelasticity in the previous chapter, the appropriate set of governing equations should consist of: seven constitutive equations, three force balance equations, one fluid diffusivity equation, and one thermal diffusivity equation. The six constitutive equations for the stress variable can be combined with the force balance equations, thus the number of final governing equations is reduced. According to the presented geological, laboratory and theoretical assumptions, the equations become considerably simplified, especially because of the limitation to the one-dimensional problem and the neglect of body forces.

There are several different ways to formulate the final governing equations for the thermal pressurization problem, since different dependent and independent variables, and various material parameters can be used.

Displacement Formulation of Force Balance Equations of Thermal Pressurization Process. Similarly as in the isothermal case (2.119), the displacement formulation of the force balance equations of thermoporoelasticity can be obtained by combining the six constitutive equations for the stress variable, given by eq. (2.46), with the force balance equations. Since the body forces are neglected during the thermal pressurization process, the force balance equations with neglected body forces (2.118) are used. By combining the constitutive relations with the force balance equations, and using the total strain definition (2.1), we obtain:

$$G \, \nabla^2 u_i \, + \, (\, G \, + \, \lambda \,) \, \frac{\partial^2 u_k}{\partial \, x_i \, x_k} \;\; = \;\; \alpha \, \frac{\partial \, p}{\partial \, x_i} \, + \, \left(\, \lambda \, + \, \frac{2 \, G}{3} \,\right) \, \alpha_b \, \frac{\partial \, T}{\partial \, x_i} \; . \quad (3.7)$$

In the case of one-dimensional problem ($\vec{u} \, = \, [\, u; 0; 0\,]$), eq. (3.7) reads:

$$(2\,G \; + \; \lambda) \; \frac{\partial^{\,2} u}{\partial\,x^{\,2}} \;\; = \;\; \alpha \, \frac{\partial\,p}{\partial\,x} \; + \; \left(\lambda \; + \; \frac{2\,G}{3} \right) \, \alpha_b \, \frac{\partial\,T}{\partial\,x} \, . \tag{3.8}$$

By applying the assumption of incompressible solid material ($\alpha \; = \; 1$), the displacement formulation of the force balance equations of the thermal pressurization process finally reads:

$$(2\,G \; + \; \lambda) \; \frac{\partial^{\,2} u}{\partial\,x^{\,2}} \;\; = \;\; \frac{\partial\,p}{\partial\,x} \; + \; \left(\lambda \; + \; \frac{2\,G}{3} \right) \, \alpha_b \, \frac{\partial\,T}{\partial\,x} \, . \tag{3.9}$$

Alternatively, it can be written in terms of the parameters β_b and β_b^v instead of λ and G, following the basic relations between poroelastic parameters (see, for example, Wang, H. (2000)):

$$\frac{\partial^{\,2} u}{\partial\,x^{\,2}} \;\; = \;\; \beta_b^v \, \frac{\partial\,p}{\partial\,x} \; + \; \frac{\beta_b^v}{\beta_b} \, \alpha_b \, \frac{\partial\,T}{\partial\,x} \, . \tag{3.10}$$

The derived relation (3.10) is in agreement with that proposed by Landau and Lifshitz (1959).

Fluid Diffusivity Equation of Thermal Pressurization Process. The fluid diffusivity equation of the thermal pressurization process can be derived from the general non-linear fluid diffusivity equation of thermoporoelasticity in the one-dimensional form, given by eq. (2.158).

To obtain it, an expression for the rate of porosity change ($\partial n/\partial t$) under conditions of constant fault-normal stress ($\sigma_n \; = \; 0$) in terms of the temperature (T) and the pore fluid pressure (p) will be derived in what follows. Subsequently, the obtained expression will be substituted for the first term in eq. (2.158).

Let us start the derivation with a relation between the Lagrangian porosity (n), the bulk volumetric strain (ε_{kk}) and the solid volumetric strain (ε_{kk}^s), which can be easily derived from the definition of the Lagrangian porostiy (2.8) by using eqs. (2.2) and (2.5):

$$n \; - \; n_0 \;\; = \;\; \varepsilon_{kk} \; - \; (1 \; - \; n_0) \; \varepsilon_{kk}^s \, . \tag{3.11}$$

Differentiating the above equation with respect to time leads to the equation:

$$\dot{n} \;\; = \;\; \dot{\varepsilon}_{kk} \; - \; (1 \; - \; n_0) \; \dot{\varepsilon}_{kk}^s \, . \tag{3.12}$$

Then, by performing the time derivatives of the equations defining the bulk volumetric strain (2.2) and the solid volumetric (2.5), the following relations are obtained,

$$\dot{\varepsilon}_{kk} = \frac{\dot{V}_b}{V_b^0} , \tag{3.13}$$

$$\dot{\varepsilon}_{kk}^s = \frac{\dot{V}_s}{V_s^0} , \tag{3.14}$$

which can be further modified by realizing that under conditions of constant fault-normal stress ($\sigma_n = 0$), any volume change (i.e., δV_b, δV_s or δV_p) can be only due to the change in the pore fluid pressure (p) or in the temperature (T). Thus we get:

$$\dot{\varepsilon}_{kk} = \frac{1}{V_b^0} \left(\frac{\partial V_b}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial V_b}{\partial T} \frac{\partial T}{\partial t} \right) , \tag{3.15}$$

$$\dot{\varepsilon}_{kk}^s = \frac{1}{V_s^0} \left(\frac{\partial V_s}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial V_s}{\partial T} \frac{\partial T}{\partial t} \right) . \tag{3.16}$$

Finally, taking into account eqs. (2.48), (2.54) and (2.69), eq. (3.15) can be rewritten in terms of the drained bulk compressibility (β_b), the solid bulk compressibility (β_s), and the solid thermal expansion coefficient (α_s):

$$\dot{\varepsilon}_{kk} = (\beta_b - \beta_s) \frac{\partial p}{\partial t} + \alpha_s \frac{\partial T}{\partial t} . \tag{3.17}$$

Similarly, eq. (3.16) can be rewritten in terms of the Lagrangian porosity in the initial state (n_0), the solid bulk compressibility (β_s), and the solid thermal expansion coefficient (α_s):

$$\dot{\varepsilon}_{kk}^s = \frac{n_0}{(1 - n_0)} \beta_s \frac{\partial p}{\partial t} + \alpha_s \frac{\partial T}{\partial t} . \tag{3.18}$$

Here, we have used Betti-Maxwell reciprocal theorem, i.e., the equality

$$\frac{\partial V_b}{\partial p} \Big|_{\sigma = 0} = \frac{\partial V_p}{\partial \sigma} \Big|_{p = 0} , \tag{3.19}$$

which postulates that the change in the bulk volume (V_b) due to the change in the pore fluid pressure (p) is equal to the change in the pore volume (V_p) due to the change in the mean total stress (σ) which is of equal value as the pore fluid pressure (p) (Detournay and Cheng, 1993).

After substituting eqs. (3.17) and (3.18) into eq. (3.12), and performing some algebraic manipulations, we obtain the desired relation for the rate of porosity change under conditions of constant fault-normal stress ($\sigma_n = 0$):

$$\dot{n} = (\beta_b - (1 + n_0) \beta_s) \frac{\partial p}{\partial t} + n_0 \alpha_s \frac{\partial T}{\partial t} . \tag{3.20}$$

We note that the obtained relation (3.20) is in agreement with that proposed by Ghabezloo and Sulem (2008a).

Finally, after the substitution of eq. (3.20) into eq. (2.158), and performing some mathematical manipulations, the desired *fluid diffusivity equation of the thermal pressurization process* is obtained in the form:

$$\frac{\partial p}{\partial t} \, (\, \beta_b \, + \, n_0 \, \beta_f \, - \, (\, 1 \, + \, n_0 \,) \, \beta_s \,) \, + \, \frac{\partial T}{\partial t} \, n_0 \, (\, \alpha_s \, - \, \alpha_f \,) \, -$$
$$- \, \frac{\partial}{\partial x} \left[\frac{k}{\mu} \frac{\partial p}{\partial x} \right] \, - \, \frac{k}{\mu} \, \beta_f \left(\frac{\partial p}{\partial x} \right)^2 \, + \, \frac{k}{\mu} \, \alpha_f \left(\frac{\partial p}{\partial x} \right) \left(\frac{\partial T}{\partial x} \right) \, = \, 0 \, . \tag{3.21}$$

An important comment should be made here. The same relation for the rate of porosity change as (3.20) would be obtained by using the force balance equation of the thermal pressurization process (3.10) together with eqs. (3.11) and (3.18) and some relevant relations between poroelastic parameters. We have used another way of derivation because it seemed more straightforward to us.

Thermal Diffusivity Equation of Thermal Pressurization Process. The one-dimensional thermal diffusivity equation of thermoporoelasticity in the general non-linear form (2.177) is directly applicable as the *thermal diffusivity equation of the thermal pressurization process*:

$$\frac{\partial}{\partial x} \left(k_{th} \frac{\partial T}{\partial x} \right) \, + \, \rho_f \, c_f \, \frac{k}{\mu} \left(\frac{\partial p}{\partial x} \right) \left(\frac{\partial T}{\partial x} \right) \, + \, n \, \alpha_f \, T \, \frac{\partial p}{\partial t} \, -$$
$$- \, \alpha_f \, \frac{k}{\mu} \, T \, \left(\frac{\partial p}{\partial x} \right)^2 \, + \, Q_s \, = \, c_e \, \frac{\partial T}{\partial t} \, . \tag{3.22}$$

After the substitution for the heat source (Q_s) in the above equation according to eq. (3.6), the thermal diffusivity equation (3.22) can be rewritten in the form:

$$\frac{\partial}{\partial x} \left(k_{th} \frac{\partial T}{\partial x} \right) \, + \, \rho_f \, c_f \, \frac{k}{\mu} \left(\frac{\partial p}{\partial x} \right) \left(\frac{\partial T}{\partial x} \right) \, + \, n \, \alpha_f \, T \, \frac{\partial p}{\partial t} \, -$$
$$- \, \alpha_f \, \frac{k}{\mu} \, T \, \left(\frac{\partial p}{\partial x} \right)^2 \, + \, f \, (\, - \sigma_n \, - \, p \,) \, \frac{\Delta V_s}{2 \, w_s} \, = \, c_e \, \frac{\partial T}{\partial t} \, ; \quad |x| \, \leq \, w_s \, ,$$
$$\frac{\partial}{\partial x} \left(k_{th} \frac{\partial T}{\partial x} \right) \, + \, \rho_f \, c_f \, \frac{k}{\mu} \left(\frac{\partial p}{\partial x} \right) \left(\frac{\partial T}{\partial x} \right) \, + \, n \, \alpha_f \, T \, \frac{\partial p}{\partial t} \, - \tag{3.23}$$
$$- \, \alpha_f \, \frac{k}{\mu} \, T \, \left(\frac{\partial p}{\partial x} \right)^2 \, = \, c_e \, \frac{\partial T}{\partial t} \, ; \quad |x| \, > \, w_s \, .$$

Simplified Governing Equations for Thermal Pressurization Process. The governing equations derived so far, given by (3.21) and (3.22), can be further simplified. First, applying the assumption of spatially constant permeability (k) and dynamic

fluid viscosity (μ), and the assumption of time constant porosity (n), the governing equations for the pore fluid pressure and temperature variations respectively read:

$$\frac{\partial p}{\partial t} \left(\beta_b + n_0 \beta_f - (1 + n_0) \beta_s \right) + \frac{\partial T}{\partial t} n_0 (\alpha_s - \alpha_f) -$$

$$- \frac{k}{\mu} \frac{\partial^2 p}{\partial x^2} - \frac{k}{\mu} \beta_f \left(\frac{\partial p}{\partial x} \right)^2 + \frac{k}{\mu} \alpha_f \left(\frac{\partial p}{\partial x} \right) \left(\frac{\partial T}{\partial x} \right) = 0, \tag{3.24}$$

$$k_{th} \frac{\partial^2 T}{\partial x^2} + \rho_f c_f \frac{k}{\mu} \left(\frac{\partial p}{\partial x} \right) \left(\frac{\partial T}{\partial x} \right) + n_0 \alpha_f T \frac{\partial p}{\partial t} -$$

$$- \alpha_f \frac{k}{\mu} T \left(\frac{\partial p}{\partial x} \right)^2 + Q_s = c_e \frac{\partial T}{\partial t}. \tag{3.25}$$

Now, using the assumption of incompressible solid material, i.e., $\beta_s = 0$, and the fact that the pore fluid thermal expansivity (α_f) is about two orders of magnitude greater than the solid thermal expansivity (α_S) (see Tab. 2), the first governing equation (3.24) can be simplified. A further simplification consists in realizing that the value of $n_0 \beta_f$ is slightly larger than the value of β_b, hence the first term in eq. (3.24) can be further simplified (see Tabs. 2 and 3). The governing equations then read:

$$\frac{\partial p}{\partial t} n_0 \beta_f - \frac{\partial T}{\partial t} n_0 \alpha_f - \frac{k}{\mu} \frac{\partial^2 p}{\partial x^2} - \frac{k}{\mu} \beta_f \left(\frac{\partial p}{\partial x} \right)^2 +$$

$$+ \frac{k}{\mu} \alpha_f \left(\frac{\partial p}{\partial x} \right) \left(\frac{\partial T}{\partial x} \right) = 0, \tag{3.26}$$

$$k_{th} \frac{\partial^2 T}{\partial x^2} + \rho_f c_f \frac{k}{\mu} \left(\frac{\partial p}{\partial x} \right) \left(\frac{\partial T}{\partial x} \right) + n_0 \alpha_f T \frac{\partial p}{\partial t} -$$

$$- \alpha_f \frac{k}{\mu} T \left(\frac{\partial p}{\partial x} \right)^2 + Q_s = c_e \frac{\partial T}{\partial t}. \tag{3.27}$$

Non-dimensionalisation of Governing Equations and Quantification of Individual Terms. The governing equations for the thermal pressurization process (3.26), (3.27) are non-linear equations. They are difficult to solve even numerically. Therefore, we transform them into a non-dimensional form and quantify the individual terms, in order to investigate whether a further simplification of them by omitting small terms can be made.

In order to transform the governing equations into a non-dimensional form, non-dimensional variables must be introduced. There are four variables appearing in the governing equations (3.26) and (3.27). Two of them are independent variables (the

time t and the position x), and two are dependent variables (the pore fluid pressure $p = p(x, t)$ and the temperature $T = T(x, t)$). The four non-dimensional variables are thus defined - the non-dimensional pressure (Π), the non-dimensional temperature (Θ), the non-dimensional time (τ), and the non-dimensional position (ξ),

$$
\begin{aligned}
\Pi &= \frac{p}{p_{ch}} , \\
\Theta &= \frac{T}{T_{ch}} , \\
\tau &= \frac{t}{t_{ch}} , \\
\xi &= \frac{x}{x_{ch}} ,
\end{aligned}
\tag{3.28}
$$

where p_{ch}, T_{ch}, t_{ch}, and x_{ch} denote, respectively, the characteristic pore fluid pressure, the characteristic temperature, the characteristic time, and the characteristic length scale of the thermal pressurization of pore fluid process. In addition, the non-dimensional Terzaghi's effective pressure (Π') is introduced, in order to put the heat source term (Q_s) into a dimensionless form,

$$
\Pi' = \frac{P'_e}{P'_{e_{ch}}} ,
\tag{3.29}
$$

where $P'_{e_{ch}}$ is the characteristic Terzaghi's effective pressure.

Values of the characteristic quantities are chosen as follows. The total slip duration at a point (t_{tot}) is taken as the characteristic time (t_{ch}) value. The characteristic pore fluid pressure (p_{ch}), the characteristic temperature (T_{ch}), and the characteristic Terzaghi's effective pressure ($P'_{e_{ch}}$) values are chosen as the time-averaged values of the pore fluid pressure (p), temperature (T), and Terzaghi's effective pressure (P'_e), respectively, during the total slip duration at a fault point (t_{tot}) (see Tabs. 3 and 1). The characteristic length scale (x_{ch}) value is assumed to be represented by the typical distance from the fault plane that is reached by the pore fluid pressure front during the total slip duration at a point, i.e., by the characteristic thickness of the hydraulic boundary layer (Lachenbruch, 1980). According to Tabs. 1 and 3, the characteristic values are

$$p_{ch} = 1.3 \times 10^8 \ Pa \ ,$$
$$T_{ch} = 10^3 \ K \ ,$$
$$x_{ch} = 10^{-2} \ m \ ,$$
$$t_{ch} = 10 \ s \ ,$$
$$P'_{e_{ch}} = 63 \times 10^6 \ Pa \ . \tag{3.30}$$

After the substitution for p, T, t, x, and Q_s in the governing equations (3.26) and (3.27) according to (3.28) we obtain:

$$
\begin{aligned}
&n_0 \ \beta_f \ \frac{\partial \ (\ p_{ch} \ \Pi \ )}{\partial \ (\ t_{ch} \ \tau \ )} \ - \ n_0 \ \alpha_f \ \frac{\partial \ (\ T_{ch} \ \Theta \ )}{\partial \ (\ t_{ch} \ \tau \ )} \ - \ \frac{k}{\mu} \ \frac{\partial^{\, 2} (\ p_{ch} \ \Pi \ )}{\partial \ (\ x_{ch} \ \xi \ )^2} \ - \\
&- \ \frac{k}{\mu} \ \beta_f \ \left\{ \frac{\partial \ (\ p_{ch} \ \Pi \ )}{\partial \ (\ x_{ch} \ \xi \ )} \right\}^2 \ + \ \frac{k}{\mu} \ \alpha_f \ \frac{\partial \ (\ p_{ch} \ \Pi \ )}{\partial \ (\ x_{ch} \ \xi \ )} \ \frac{\partial \ (\ T_{ch} \ \Theta \ )}{\partial \ (\ x_{ch} \ \xi \ )} \ = \ 0 \ ,
\end{aligned}
\tag{3.31}
$$

$$
\begin{aligned}
&k_{th} \ \frac{\partial^2 \ (\ T_{ch} \ \Theta \ )}{\partial \ (\ x_{ch} \ \xi \ )^2} \ + \ \rho_f \ c_f \ \frac{k}{\mu} \ \frac{\partial \ (\ p_{ch} \ \Pi \ )}{\partial \ (\ x_{ch} \ \xi \ )} \ \frac{\partial \ (\ T_{ch} \ \Theta \ )}{\partial \ (\ x_{ch} \ \xi \ )} \ - \ c_e \ \frac{\partial \ (\ T_{ch} \ \Theta \ )}{\partial \ (\ t_{ch} \ \tau \ )} \ + \\
&+ \ n_0 \ \alpha_f \ (\ T_{ch} \ \Theta \ ) \ \frac{\partial \ (\ p_{ch} \ \Pi \ )}{\partial \ (\ t_{ch} \ \tau \ )} \ - \ \alpha_f \ \frac{k}{\mu} \ (\ T_{ch} \ \Theta \ ) \ \left\{ \frac{\partial \ (\ p_{ch} \ \Pi \ )}{\partial \ (\ x_{ch} \ \xi \ )} \right\}^2 \ + \\
&\hspace{6cm} + \ \frac{f \ \Delta V_s}{2 \ w_s} \ P'_{e_{ch}} \ \Pi' \ = \ 0 \ .
\end{aligned}
\tag{3.32}
$$

Here, the heat source definition (3.5) was used to make the substitution for Q_s possible.

Now, we will introduce an expression for the characteristic time (t_{ch}) in terms of another parameters or characteristic quantities. In order to derive such expression, every two linear terms in the governing equations which contain the characteristic time are put into equality, from which the relation for t_{ch} is thus found (for each couple of terms). Finally, the relevant values from Tabs. 2 and 3 are substituted into the obtained expressions. The expression for which the calculated characteristic time value is about 10 s (which is the value corresponding to the anticipation, see (3.30)), is chosen as the proper expression for the characteristic time (t_{ch}). Thereby the expression is obtained by putting the first and the third term on the l.h.s. of eq. (3.31) into equality. It reads:

$$t_{ch} = n_0 \ \beta_f \ \frac{\mu}{k} \ x_{ch}^2 \ . \tag{3.33}$$

Taking into account eq. (2.101), the above relation can be rewritten in terms of the hydraulic diffusivity (α_{hy}). Under conditions of time constant porosity (n), eq. (3.33) in terms of α_{hy} reads:

$$t_{ch} = \frac{x_{ch}^2}{\alpha_{hy}} \,. \tag{3.34}$$

Now, the obtained relation (3.33) is substituted for the characteristic time (t_{ch}) in the governing equations (3.31), (3.32). By multiplying eq. (3.31) with $(\mu\, x_{ch}^2) / (k\, p_{ch})$ and eq. (3.32) with $x_{ch}^2 / (k_{th}\, T_{ch})$, using the thermal diffusivity (α_{th}) definition (2.114), and eq. (2.101) for the hydraulic diffusivity (α_{hy}), the dimensionless governing equations are obtained in the form:

$$\frac{\partial\,\Pi}{\partial\,\tau} \;-\; \frac{\alpha_f\, T_{ch}}{\beta_f\, p_{ch}} \frac{\partial\,\Theta}{\partial\,\tau} \;-\; \frac{\partial^2\Pi}{\partial\,\xi^2} \;-\; \beta_f\, p_{ch} \left(\frac{\partial\,\Pi}{\partial\,\xi}\right)^2 \;+$$
$$+ \;\; \alpha_f\, T_{ch} \left(\frac{\partial\,\Pi}{\partial\,\xi}\right) \left(\frac{\partial\,\Theta}{\partial\,\xi}\right) \;\; = \;\; 0 \,, \tag{3.35}$$

$$\frac{\partial^2\Theta}{\partial\,\xi^2} \;+\; Pe \left(\frac{\partial\,\Pi}{\partial\,\xi}\right)\left(\frac{\partial\,\Theta}{\partial\,\xi}\right) \;-\; \frac{\alpha_{hy}}{\alpha_{th}} \frac{\partial\,\Theta}{\partial\,\tau} \;+\; \frac{n_0\, \alpha_f\, p_{ch}\, \alpha_{hy}}{k_{th}} \,\Theta\, \frac{\partial\,\Pi}{\partial\,\tau} \;-$$
$$-\; \frac{\alpha_f\, k\, p_{ch}^2}{\mu\, k_{th}} \,\Theta \left(\frac{\partial\,\Pi}{\partial\,\xi}\right)^2 \;+\; \frac{f\,\Delta V_s\, P'_{e_{ch}}\, x_{ch}^2}{2\, w_s\, k_{th}\, T_{ch}} \,\Pi' \;\; = \;\; 0 \,. \tag{3.36}$$

Here, Pe denotes the so-called *Peclet number* (Lee and Delaney, 1987), defined as

$$Pe \;\; = \;\; \frac{\rho_f\, c_f\, k\, p_{ch}}{\mu\, k_{th}} \,. \tag{3.37}$$

Let us make the following denotations of the dimensionless groups of quantities in eqs. (3.35) and (3.36):

$$A \;\; = \;\; \frac{\alpha_f\, T_{ch}}{\beta_f\, p_{ch}} \,,$$

$$B \;\; = \;\; \beta_f\, p_{ch} \,,$$

$$C \;\; = \;\; \alpha_f\, T_{ch} \,,$$

$$D \;\; = \;\; \frac{\alpha_{hy}}{\alpha_{th}} \,, \tag{3.38}$$

$$E \;\; = \;\; \frac{n_0\, \alpha_f\, p_{ch}\, \alpha_{hy}}{k_{th}} \,,$$

$$F \;\; = \;\; \frac{\alpha_f\, k\, p_{ch}}{\mu\, k_{th}} \,,$$

$$G \;\; = \;\; \frac{f\,\Delta V_s\, P'_{e_{ch}}\, x_{ch}^2}{2\, w_s\, k_{th}\, T_{ch}} \,.$$

According to (3.38), the governing equations (3.35) and (3.36) can be rewritten as

$$\frac{\partial \Pi}{\partial \tau} - A \frac{\partial \Theta}{\partial \tau} - \frac{\partial^2 \Pi}{\partial \xi^2} - B \left(\frac{\partial \Pi}{\partial \xi} \right)^2 +$$
$$+ C \left(\frac{\partial \Pi}{\partial \xi} \right) \left(\frac{\partial \Theta}{\partial \xi} \right) = 0 , \tag{3.39}$$

$$\frac{\partial^2 \Theta}{\partial \xi^2} + Pe \left(\frac{\partial \Pi}{\partial \xi} \right) \left(\frac{\partial \Theta}{\partial \xi} \right) - D \frac{\partial \Theta}{\partial \tau} + E \Theta \frac{\partial \Pi}{\partial \tau} -$$
$$- F \Theta \left(\frac{\partial \Pi}{\partial \xi} \right)^2 + G \Pi' = 0 . \tag{3.40}$$

The individual terms in eq. (3.39) are respectively from left to right of order 1, A, 1, B, and C. Similarly, the terms in eq. (3.40) are of order 1, Pe, D, E, F, and G. To quantify the order of the individual terms in the governing equations, a substitution of the relevant values into (3.38) according to Tabs. 1, 2, and 3 is made,

$$A = \frac{(2.5 \times 10^{-3} \ K^{-1}) \, (10^3 \ K)}{(4.5 \times 10^{-9} \ Pa^{-1}) \, (1.3 \times 10^8 \ Pa)} \approx 4.2$$

$$B = (4.5 \times 10^{-9} \ Pa^{-1}) \, (1.3 \times 10^8 \ Pa) \approx 0.6$$

$$C = (2.5 \times 10^{-3} \ K^{-1}) \, (10^3 \ K) \approx 2.5$$

$$Pe = \frac{(340 \ kg \ m^{-3}) \, (2.3 \times 10^3 \ J \ kg^{-1} \ K^{-1}) \, (10^{-19} \ m^2) \, (1.3 \times 10^8 \ Pa)}{(5.7 \times 10^{-5} \ Pa \ s) \, (1.35 \ J \ s^{-1} \ m^{-1} \ K^{-1})}$$
$$\approx 0.13$$

$$D = \frac{10^{-5} \ m^2 \ s^{-1}}{0.5 \times 10^{-6} \ m^2 \ s^{-1}} \approx 20 \tag{3.41}$$

$$E = \frac{0.04 \ (2.5 \times 10^{-3} \ K^{-1}) \, (1.3 \times 10^8 \ Pa) \, (10^{-5} \ m^2 \ s^{-1})}{1.35 \ J \ s^{-1} \ m^{-1} \ K^{-1}} \approx 0.1$$

$$F = \frac{(2.5 \times 10^{-3} \ K^{-1}) \, (10^{-19} \ m^2) \, (1.69 \times 10^{16} \ Pa^2)}{(5.7 \times 10^{-5} \ Pa \ s) \, (1.35 \ J \ s^{-1} \ m^{-1} \ K^{-1})} \approx 5.46 \times 10^{-2}$$

$$G = \frac{0.45 \ (1 \ m \ s^{-1}) \, (63 \times 10^6 \ Pa) \, (10^{-4} \ m^2)}{2 \ (1 \times 10^{-3} \ m) \, (1.35 \ J \ s^{-1} \ m^{-1} \ K^{-1}) \, (10^3 \ K)} \approx 10^3 .$$

The corresponding terms in the governing equations (3.39), (3.40) are thus of the following orders:

$$A \propto 1 , \quad B \propto (0.1 \ ; \ 1) , \quad C \propto 1 , \quad Pe \propto 0.1 \tag{3.42}$$
$$D \propto 10 , \quad E \propto 0.1 , \quad F \propto 0.1 , \quad G \propto 10^3 .$$

The first and the most unexpected result of the analysis regarding the governing equation for the pore fluid pressure variation (3.39) is that at least one non-linear term cannot be omitted in the equation. Commonly, all the non-linear terms are omitted and the linearized form of (3.39) is used as the governing equation for the thermal pressurization process. However, our analysis shows that the non-linear term containing the coefficient C is of the same order as the linear terms in the equation. Furthermore, the non-linear term containing the coefficient B is either of the same order or by one order smaller than the linear terms. Because of the uncertainty in values of various parameters used to determine the order of the terms, we do not neglect either of the non-linear terms. Hence, the dimensionless governing equation for the pore fluid pressure variation is the original non-linear eq. (3.39), now rewritten with the terms arranged according to their order:

$$\frac{\partial \Pi}{\partial \tau} - A \frac{\partial \Theta}{\partial \tau} - \frac{\partial^2 \Pi}{\partial \xi^2} + C \left(\frac{\partial \Pi}{\partial \xi}\right)\left(\frac{\partial \Theta}{\partial \xi}\right) - B \left(\frac{\partial \Pi}{\partial \xi}\right)^2 = 0 . \quad (3.43)$$

Here, the first four terms are of the same order. The second and third linear terms are the source and the transport terms, respectively. The terms containing the coefficient C and B represent the change in pore fluid mass respectively due to the the thermal expansion and due to the compression of the pore fluid along the flow path (Mase and Smith, 1985).

The second result of the performed dimensional analysis is that the nonlinear term containing Peclet number (Pe), and the coupling terms containing the coefficients E and F can be neglected in the governing equation for the temperature variation (3.40), since they are of relatively small orders. Consequently, the non-linear governing equation (3.40) becomes linear. Physically, the omission of the term containing Peclet number means that the convective heat transport is neglected. The omission of the terms containing coefficients E and F means that the heating due to the reversible work done on the fluid (Mase and Smith, 1985) is not taken into account. The resultant dimensionless governing equation for the temperature variation hence reads:

$$G \Pi' - D \frac{\partial \Theta}{\partial \tau} + \frac{\partial^2 \Theta}{\partial \xi^2} = 0 . \quad (3.44)$$

Here, the terms are arranged according to their order. The first term, which represents the heat source, is the highest order term. The second term represents the heat storage.

76

The last term is the smallest order term in the equation and is often referred to as the transport term (Lachenbruch, 1980). It represents the conductive transport of heat.

The obtained linearized equation (3.44) has a very advantageous form. If a planar heat source confined to the fault plane was used in the model (instead of the proposed more realistic one that is confined to the finite-thickness PSZ), the heat source can be introduced through a boundary condition. Hence, the heat source term ($G\,\Pi'$) vanishes in the equation, hence the equation becomes uncoupled from the second governing equation for the pore fluid pressure variation (3.43). Consequently, the equation for the temperature variation can be solved independently from the pressure diffusion equation. Its solution is then used to solve the governing equation for the pore fluid pressure variation.

Final Governing Equations of Thermal Pressurization of Pore Fluid Process.
Following the preceding dimensional analysis and the derived governing equations given by (3.26) and (3.27), we propose the following final governing equations for the thermal pressurization of pore fluid process,

$$
\frac{\partial p}{\partial t}\,n_0\,\beta_f \;-\; \frac{\partial T}{\partial t}\,n_0\,\alpha_f \;-\; \frac{k}{\mu}\,\frac{\partial^2 p}{\partial x^2} \;-\; \frac{k}{\mu}\,\beta_f\left(\frac{\partial p}{\partial x}\right)^2 \;+
$$
$$
+\;\frac{k}{\mu}\,\alpha_f\left(\frac{\partial p}{\partial x}\right)\left(\frac{\partial T}{\partial x}\right) \;=\; 0\,, \tag{3.45}
$$

$$
k_{th}\,\frac{\partial^2 T}{\partial x^2} \;+\; Q_s \;=\; c_e\,\frac{\partial T}{\partial t}\,, \tag{3.46}
$$

where the heat source is non-zero ($Q_s \neq 0$) only within the PSZ ($|x| \leq w_s$).

The second governing equation (3.46), which is the equation for the temperature variation, is identical to the commonly used equation in the models of the thermal pressurization of pore fluid process during earthquake slip (Andrews, 2002), (Bizzarri and Cocco, 2006a), (Rempel and Rice, 2006), (Rice, 2006), (Segall and Rice, 2006). However, the first governing equation (3.45), which is the equation for the pore fluid pressure variation, differs from the commonly used equation - it contains additionally two non-linear terms, which cannot be generally neglected according to the presented dimensional analysis.

3.2.6 Primary Shortcomings of the Presented Model

1. Plasticity is not taken into account (only the elasticity is considered).

   Especially the local stresses at the propagating rupture front may be sufficiently high to cause irreversible changes in material properties and inelastic deformations which are detected in the fault gouge materials (Poliakov et al., 2002), (Andrews, 2005), (Rice and Cocco, 2006). The effect becomes considerable especially as the rupture propagation velocity approaches the Rayleigh velocity (Rice and Cocco, 2006). The high rupture front stresses may cause the dilatancy of the material (inelastic pore volume increase) leading to the significant increase in permeability (due to its strong dependence on porosity), thus reducing the pore fluid pressure and increasing the material strength (Viesca et al., 2008). The studies, however, suggest that dilatancy is important during a nucleation phase of the rupture (when the highest stress concentrations exist at the rupture front), but it has a negligible effect after a steady slipping is developed (Segall and Rice, 1995). Then, the frictional heating, possibly accompanied by the thermal pressurization of pore fluid, is the primary mechanism acting on the fault. Since the dilatancy seems to have a negligible effect during the thermal pressurization process, omitting the dilatancy in the physical model should not distort the results significantly.

2. The material is assumed to be macroscopically homogeneous, i.e., the values of material parameters are spatial constants.

   The validity of this assumption is questionable particularly due to the large spatial variations in the permeability value in the fault-normal direction. However, since the thermal pressurization process effectively occurs only at relatively small fault-normal length scales, such assumption might be admissible.

3. Values of material parameters are assumed to be constant during the earthquake slip.

   This is surely not true especially for the porosity (n), permeability (k), pore fluid density (ρ_f), pore fluid viscosity (μ), pore fluid compressibility (β_f), pore fluid thermal expansivity (α_f), drained bulk compressibility (β_b), and hydraulic diffusivity (α_{hy}), which are the most sensitive parameters to changes in the pore fluid pressure and temperature. The assumption might hold and the usage of time-averaged

78

values might be admissible under conditions that the variations in parameters are relatively small and linear. However, there is some evidence that the variations becomes non-linear at high pressures and temperatures of interest (e.g., the pore fluid compressibility (β_f) variation with pressure becomes considerably non-linear at the temperatures higher than about $500\ K$ (Marshall, 2008)). Nevertheless, further experiments are needed in order to incorporate the variations properly into the physical model.

4. Anisotropy is not taken into account.

   The presented analysis is based both on the assumption that the bulk material is isotropic, and that the poroelastic response of the material is purely isotropic. The latter assumption means that the poroelastic response of the material follows the theory of poroelasticity introduced by Biot (1941), which predicts that for an isotropic material, only the changes in the normal stress components (and not those in the shear stress components) can induce a pore fluid pressure change or cause a volumetric deformation of the rock. However, there are some theoretical investigations and laboratory experiments suggesting that a pore fluid pressure response to the applied shear stress is possible, with a dominant elastic response, and increasing with the increasing applied shear stress (Skempton, 1954), (Wang, 1997), (Lockner and Stanchits, 2002). It appears to be the result of a stress-induced anisotropy of the material. The first assumption of the bulk material isotropy is the more inaccurate one, since several geologic observations imply that fault zone rocks have typically anisotropic material properties. Especially the anisotropy in the permeability is significant - the permeability in the fault-parallel direction can be even 3 orders of magnitude higher than that in the fault normal direction (Faulkner and Rutter, 2005).

5. The model is kinematic.

   We do not consider the dynamic effects, e.g., the effect of the fault strength (τ) variations on the slip rate (ΔV_s), and thus on the heat source (Q_s), is not taken into account.

6. Chemical effects are not considered.

Various studies suggest that the effect of coseismic chemical interactions between the pore fluid and the solid material of the rock is non-negligible, thus should be incorporated into the physical model of the thermal pressurization process. Particularly the thermally induced dehydration of the rock hydrous minerals (e.g., clay-rich minerals) possibly acts as an additional source term in the governing equation for the pore fluid pressure variation of the thermal pressurization process (Hirose and Bystricky, 2007), (Brantut et al., 2008). Furthermore, the chemical reactions may consume a significant amount of the frictional heat generated during earthquake slip, and thereby suppress the temperature rise (and hence suppress melting). Under sufficiently high concentrations of reactive materials, the frictional heat consumption could possibly reach 50 % of the frictional heat generated during slip (Hamada et al., 2009). The chemical effects can also contribute to the explanation of the heat flow paradox (Jacobs et al., 2006). Thus the chemical effects might be taken into account in the future thermal pressurization models.

7. Pure water is assumed to be the pore fluid. Moreover, we assume that the water exists in a single phase.

   The first assumption is surely not satisfied under real fault conditions. The pore fluid typically consists of a water solution containing Na, Cl^{-1}, Sr, SO_4^{2-}, among others, which could possibly change the pore fluid behavior and its chemical interactions with the rock during the thermal pressurization process. The second assumption on the water phase is also a matter of debate. For example, under hydrostatic conditions, assuming the normal geothermal gradient ($30\,K$ per kilometer of depth) and the typical seismogenic depth ($7\,km$), the liquid water should undergo a phase change into a supercritical state during the thermal pressurization of pore fluid process. The critical point of water ($647\,K$, $22\,MPa$) is exceeded during the large temperature and pore fluid pressure changes (Chaplin, 2009). The phase change into the supercritical state could significantly affect some of the water properties. For example, the supercritical water is highly compressible, implying that even small changes in pressure can induce large changes in the water density (ρ_f), thus affecting its viscosity (μ) and solvation properties in chemical reactions with the rock. Moreover, experiments performed with granite show a rapid increase

in permeability (k) when the critical temperature of water is reached ($647\,K$), which may be due to a micro-fracture network generation under supercritical water conditions (Takahashi et al., 2003).

3.3 Main Points of Controversy and Open Questions

Although the thermal pressurization of pore fluid process during earthquake slip has been known and studied for more than a quarter-century, there are still some controversial issues on it. Particularly the following two points are still a matter of debate:

1. The effective stress principle lacks a laboratory verification at coseismic slip rates.
2. The connectivity of the porous rock space can be lost during the interseismic period, for example due to the mineralization of the rock (Rice, 2006). However, although in the case that the hypothesis holds, there are some studies suggesting that coseismic stress concentrations at the rupture tip are possibly high enough to induce a reconnection of the pores (Poliakov et al., 2002). If the rock connectivity was corrupted, the classical theory of poroelasticity would not be applicable since it is built on the assumption of the porous space interconnection.

4 Conclusions

In this thesis, we

- summarized the most important dynamic fault weakening mechanisms, briefly characterized them, and assessed their possible contributions to the dynamic fault weakening during tectonic earthquakes,
- presented an introductory text on the theory of thermoporoelasticity, starting with a detailed development of the theory of linear poroelasticity, following with its extension to a thermoporoelastic case, and including a detailed derivation of the general non-linear governing equations of thermoporoelasticity,
- summarized the latest geological, laboratory, and theoretical results regarding the thermal pressurization of pore fluid process,
- proposed a set of appropriate values of the parameters relevant for the thermal pressurization of pore fluid process,
- proposed a modified physical model of the thermal pressurization of pore fluid process, following the recent field results and the theory of thermoporoelasticity.

By modifying the general non-linear governing equations of thermoporoelasticity according to the properties of the physical model of the thermal pressurization of pore fluid process, we obtained the governing equations adequate for the process. Then, we transformed them into a non-dimensional form, and quantified the individual terms. The non-dimensionalisation shows that

- the governing equation for the temperature variations can be used in its linearized form,
- the governing equation for the pore fluid pressure variations, however, cannot be used in its linearized form, since at least one non-linear term cannot be neglected in the equation, as being of the same order of magnitude as the linear terms in the equation.

Commonly, both governing equations are applied in their linear form. Our analysis, however, implies that one of the equations should be used in its original non-linear form. We admit that our results may be slightly inaccurate, as a consequence of the

considerable uncertainty in values of various model parameters. Nevertheless, just due to the latter fact, we guess that the non-linear form of the governing equation for the pore fluid pressure variations should be used as the proper governing equation for the thermal pressurization process.

References

Archard, J. F., 1958-1959. The temperature of rubbing surfaces. *Wear* 2, 438–455.

Andrews, D. J., 2002. A fault constitutive relation accounting for thermal pressurization of pore fluid. *J. Geophys. Res.* 107B12, 2363, doi:10.1029/2002JB001942.

Andrews, D. J., 2005. Rupture dynamics with energy loss outside the slip zone. *J. Geophys. Res.* 110(B1), 1307.

Arsenin, V. J., 1977. Matematická fyzika. *Alfa*.

Batchelor, G. H., 1967. An Introduction to Fluid Dynamics. *Cambridge University Press*.

Bear, J., 1988. Dynamics of Fluids in Porous Media. *Courier Dover Publications*.

Beeler, N. M., and T. E. Tullis, 2003. Constitutive relationships for fault strength due to flash-heating. *SCEC Annual Meeting Proceedings and Abstracts* XIII, p.66.

Bejan, A., I. Dinçer, S. Lorente, A. F. Miguel, and A. H. Reis, 2004. Porous and complex flow structures in modern technologies. *Springer*, ISBN 0387202250.

Berryman, J. G., 1992. Effective stress for transport properties of inhomogeneous porous rock. *Journal of Geophysical Research* 97, 17409-17424.

Biot, M. A., 1941. General theory of three-dimensional consolidation. *Journal of Applied Physics* 12, 155-164.

Biot, M. A., 1955. Theory of elasticity and consolidation for a porous anisotropic solid. *Journal of Applied Physics* 26, 182-185.

Biot, M. A., 1956. General solutions of the equations of elasticity and consolidation for a porous material. *J. Appl. Mech., Trans. ASME* 78, 91-96.

Biot, M. A., 1956. Thermoelasticity and irreversible thermodynamics. *Journal of Applied Physics* 27, 240-253.

Biot, M. A., 1962. Theory of propagation of elastic waves in a fluid-saturated porous solid; part I: low frequency range and part II: higher frequency range. *Journal of the Acoustical Society of America* 28, 168-178 and 179-191.

Biot, M. A., 1962. Mechanics of deformation and acoustic propagation in porous media. *Journal of Applied Physics* 33, 1482—1498.

Biot, M. A., 1973. Nonlinear and semilinear rheology of porous solids. *J. Geophys. Res.* vol.78, no.23, 4924-4937.

Biot, M. A. and D. G. Willis, 1957. The elastic coefficients of the theory of consolidation. *J. Appl. Mech.* 24, 594-601.

Bizzarri, A. and M. Cocco, 2003. Slip-weakening behavior during the propagation of dynamic ruptures obeying to rate- and state-dependent friction laws. *J. Geophys. Res.* 108, 2373.

Bizzarri, A. and M. Cocco, 2006. A thermal pressurization model for the spontaneous dynamic rupture propagation on a three-dimensional fault: 1. Methodological approach. *J. Geophys. Res.* 111, B05303, doi:10.1029/2005JB003862.

Bizzarri, A. and M. Cocco, 2006. A thermal pressurization model for the spontaneous dynamic rupture propagation on a three-dimensional fault: 2. Traction evolution and dynamic parameters. *J. Geophys. Res.*111, B05304, doi:10.1029/2005JB003864.

Bowden, F. P. and D. Tabor, 1950. The Friction and Lubrication of Solids. *Clarendon Press, Oxford*, Part I, 52-57.

Brantut, N., A. Schubnel, and J. Corvisier, 2008. Thermo-Chemical Pressurization of Fault Gouges During Coseismic Slip. *AGU, Fall Meeting 2008, abstract.*

Brodsky, E. E. and H. Kanamori, 2001. Elastohydrodynamic lubrication of faults. *J. Geophys. Res.* 106, 16357–16374.

Brune, J. N., S. Brown, and P. A. Johnson, 1993. Rupture mechanism and interface separation in foam rubber models of earthquakes: A possible solution to the heat flow paradox and the paradox of large overthrusts. *Tectonophysics* 218, 59-67.

Burridge, R. and L. Knopoff, 1967. Model and theoretical seismicity. *Bull. Seismol. Soc. Am.* 57, 341-371.

Cardwell, R. K., D. S. Chinn, G. F. Moore, and D. L. Turcotte, 1978. Frictional heating on a fault zone with finite thickness. *Geophys J. R. Astron. Soc.* 52, 525–530.

Chaplin, M., 2009. Available at `http://www.lsbu.ac.uk/water/phase.html`.

Charlez, P. A., 1991. Rock Mechanics, vol. 1, Theoretical Fundamentals. *Editions Technip, Paris.*

Chester, J. S., F. M. Chester, and A. K. Kronenberg, 2005. Fracture surface energy of the Punchbowl Fault, San Andreas System. *Nature* 437, 133–136.

Cocco, M. and A. Bizzarri, 2004. Dynamic fault weakening caused by thermal pressurization in an earthquake model governed by rate- and state-dependent friction. *EOS, Trans. AGU* 85, Fall Meeting Suppl., Abstract T22A-06.

Cocco, M., P. Spudich, and E. Tinti, 2006. On the mechanical work absorbed on faults during earthquake ruptures. Revised Version for the AGU Monograph on *Radiated Energy and the Physics of Earthquakes Faulting*, A. McGarr, R. Abercrombie, H. Kanamori and G. di Toro, Eds.

Coussy, O., 2004. Poromechanics *John Wiley and Sons*

Delaney, P. T., 1982. Rapid intrusion of magma into wet rocks: Groundwater flow due to pore pressure increases. *J. Geophys. Res.* 87, 7739–7756.

Detournay, E. and A. H.-D. Cheng, 1993. Fundamentals of poroelasticity, Chapter 5 in *Comprehensive Rock Engrg., vol. II., Pergamon Press*, J. Hudson, ed., 113-169.

Di Toro, G., D. L. Goldsby, and T. E. Tullis, 2004. Friction falls toward zero in quartz rock as slip velocity approaches seismic rates. *Nature* 427, 436–439.

Di Toro, G., T. Hirose, S. Nielsen, G. Pennacchioni, and T. Shimamoto, 2006. Natural and experimental evidence of melt lubrication of faults during earthquakes. *Science* 311, 647-649.

Engineeringtoolbox, 2009. Available at `http://www.engineeringtoolbox.com/water-thermal-properties-d_162.html`.

Faulkner, D. and E. Rutter, 2005. On the maintenance of high fluid pressures in large fault zones. *Geophysical Research Abstracts*, vol. 7.

Fauvel, E., Ch. Joussot-Dubien, P. Guichardon, G. Charbit, F. Charbit, and S. Sarrade, 2004. A double-wall reactor for hydrothermal oxidation with supercritical water flow across the inner porous tube. *The Journal of supercritical fluids* 28, 47-56.

Fialko, Y., 2004. Temperature fields generated by the elastodynamic propagation of shear cracks in the Earth. *J. Geophys. Res.* 109, B01303, doi: 10.1029/2003JB002497.

Foster, R. J., 1985. Geology (Fifth Edition). *Charles E. Merrill Publishing Company.*

Ghabezloo, S. and J. Sulem, 2008. Stress dependent thermal pressurization of a fluid-saturated rock. *Rock Mechanics and Rock Engineering* 42 (1), 1-24.

Ghabezloo, S. and J. Sulem, 2008. Experimental study of thermal pressurization phenomenon in geomaterials. *Alert Workshop 2008*, Abstract.

Goldsby, D. L., and T.E. Tullis, 2002. Low frictional strength of quartz rocks at subseismic slip rates. *Geophys. Res. Lett.*.

Gueguen, Y. and M. Bouteca, 2004. Mechanics of fluid saturated rocks. *International Geophysics Series*, vol. 89. *Elsevier Academic Press.*

Hamada, Y., T. Hirono, W. Tanikawa, W. Soh, and S.-R. Song, 2009. Energy taken up by co-seismic chemical reactions during a large earthquake: An example from the 1999 Taiwan Chi-Chi earthquake. *Geophys. Res. Lett.* 36.

Harris, R. A. and S. M. Day, 1997. Effects of a low - velocity zone on a dynamic rupture. *Bull. Seism. Soc. Am.* 87, 1267-1280.

Heaton, T. H., 1990. Evidence for and implications of self-healing pulses of slip in earthquake rupture. *Phys. Earth. Planet. Inter.* 64, 1–20.

Hirose, T. and M. Bystricky, 2007. Extreme dynamic weakening of faults during dehydration by coseismic shear heating. *Geophys. Res. Lett.* 34, L14311.

Jacobs, J. R., J. P. Evans, and P. T. Kolesar, 2006. Energentics of chemical alteration in fault zones and its relationship to the seismic cycle. *Geophysical monograph - AGU*, Vol. 170, 181-191.

Jeffreys, H., 1942. On the mechanics of faulting. *Geol. Mag.* 79, 291–295.

Kanamori, H. and E. E. Brodsky, 2001. The physics of earthquakes. *Phys. Today* 54, 34-39.

Kanamori, H. and T. H. Heaton, 2000. Microscopic and macroscopic physics of earthquakes. *Geophys. Monogr. Ser., AGU* 120, 147–163.

Kirkpatrick, J. D., Z. K. Shipton, and C. Persano, 2009. Pseudotachylytes; rarely generated, rarely preserved, or rarely reported? *Bull. Seism. Soc. Am.* 99, no. 1, 382-388.

Lachenbruch, A. H., 1980. Frictional heating, fluid pressure, and the resistance to fault motion. *J. Geophys. Res.* 85, 6097–6122.

Landau, I. and E. Lifshitz, 1959. Theory of Elasticity. Translated from Russian by J. Sykes and W. Reid. *Addison-Wesley, Reading, Mass..*

Lapusta, N. and J. R. Rice, 2003. Low-Heat and Low-Stress Fault Operation in Earthquake Models of Statically Strong but Dynamically Weak Faults. *AGU, Fall Meeting 2003 abstract.*

Lee, Tien-Chang, P. T. Delaney, 1987. Frictional heating and pore pressure rise due to a fault slip. *Geophysical Journal International* 88, 569-591.

Lockner, D., R. Summers, and J. Byerlee, 1986 Effects of temperature and sliding rate of frictional strength of granite. *Pure Appl. Geophys.* 124, 445–469.

Lockner, D., H. Naka, H. Tanaka, H. Ito, and R. Ikeda, 2000 Permeability and strength of core samples from the Nojima fault of the 1995 Kobe earthquake. In *Proceedings of the International Workshop on the Nojima Fault Core and Borehole Data Analysis*, ed. H. Ito et al., *U.S. Geol. Surv. Open File Rep.* 00-129, 147–152.

Lockner, D. A., and S. A. Stanchits, 2002 Undrained poroelastic response of sandstones to deviatoric stress change. *J. Geophys. Res.* 107(B12), 2353.

Mair, K. and C. J. Marone, 1999. Friction of simulated fault gouge for a wide range of slip velocities and normal stresses. *J. Geophys. Res.* 104, 28899-28914.

Mair, K. and C. J. Marone, 2000. Shear heating in granular layers. *Pure Appl. Geophys.* 157, 1847–1866.

Marone, C. J., 1998. Laboratory-derived friction laws and their application to seismic faulting. *Ann. Revs. Earth and Plan. Sci.* 26, 643-696.

Marshall, S. L., 2008. Nonlinear Pressure Diffusion in Flow of Compressible Liquids Through Porous Media. *Transport in Porous Media* 77, 431-446.

Mase, C. W. and L. Smith, 1985. Pore-fluid pressures and frictional heating on a fault surface. *Pure Appl. Geophys.* 122, 583–607.

Mase, C. W. and L. Smith, 1987. Effects of frictional heating on the thermal, hydrologic, and mechanical response of a fault. *J. Geophys. Res.* 92, 6249–6272.

McKenzie, D. P., and J. N. Brune, 1972. Melting on fault planes during large earthquakes. *Geophys. J. R. Astron. Soc.* 29, 65–78.

McTigue, D. F., 1986. Thermoelastic response of fluid-saturated porous rock. *J. Geophys. Res.* 91, 9533–9542.

Melosh, H. J., 1996. Dynamic weakening of faults by acoustic fluidization. *Nature* 397, 601–606.

Mizoguchi, K., T. Hirose, and T. Shimamoto, 2000. Permeability structure of Nojima fault: Analysis of Funaki outcrop in Hohundan, Tsuna-gun, Hyogo Prefecture (in Japanese). *Earth Mon. Extra* 31, 58–65.

Mizoguchi, K., and T. Shimamoto, 2004. Dramatic slip weakening of Nojima fault gouge at high-velocities and its implication for dynamic fault motion. *Eos Trans. AGU* 85(47), Fall Meet. Suppl., Abstract T23A-0559.

Moczo, P., J. Kristek, M. Galis, P. Pazak, and M. Balazovjech, 2007. The finite-difference and finite-element modeling of seismic wave propagation and earthquake motion. *Acta Physica Slovaca* vol.57, no.2, 177-406.

Morrow, C., J. Solum, S. Tembe, D. Lockner, and T.-F. Wong, 2007. Using drill cutting separates to estimate the strength of narrow shear zones at SAFOD. *Geophys. Res. Lett.* 34.

Noda, H., 2004. Numerical simulation of rupture propagation with thermal pressurization based on measured hydraulic properties: Importance of deformation zone width. *Eos Trans. AGU* 85(47), Fall Meet. Suppl., Abstract T22A-08.

Noda, H., and T. Shimamoto, 2005. Thermal pressurization and slip weakening distance of a fault: An example of the Hanaore fault, southwest Japan. *Bull. Seismol. Soc. Am.* 95, 1224–1233.

Noda, H., E. M. Dunham, and J. R. Rice, 2008 Earthquake ruptures with thermal weakening and the operation of major faults at low overall stress levels. *J. Geophys. Res.- Solid Earth*, 2008JB006143.

NIST WebBook Chemie, 2009. Available at `http://webbook.nist.gov/`.

Nuth, M., and L. Laloui, 2008. Effective stress concept in unsaturated soils: Clarification and validation of a unified framework. *Int. J. Numer. Anal. Meth. Geomech.* 32, 771–801.

Palciauskas, V. V., and P. A. Domenico, 1982. Characterization of drained and undrained response of thermally loaded repository rocks. *Water Resour. Res.*18, 281-290.

Papanastasiou, T. C., G. C. Georgios, and A. N. Alexandrou, 2000. Viscous Fluid Flow. *CRC Press.*

Poliakov, A. N. B., R. Dmowska, and J. R. Rice, 2002. Dynamic shear rupture interactions with fault bends and off-axis secondary faulting. *J. Geophys. Res.* 107(B11), 2295.

Prakash, V., 2004. Pilot studies to determine the feasibility of using new experimental techniques to measure sliding resistance at seismic slip rates. *Annu. Prog. Rep., SCEC.*

Prakash, V., and F. Yuan, 2004. Results of a pilot study to investigate the feasibility of using new experimental techniques to measure sliding resistance at seismic slip rates. *Eos Trans. AGU* 85(47), Fall Meet. Suppl., Abstract T21D-02.

Raleigh, C. B., and J. Everden, 1981. Case for low deviatoric stress in the lithosphere, in *The Mechanical Behavior of Crustal Rocks*, edited by N. L. Carter et al.. *Geophys. Monogr. Ser., AGU, Washington, D. C* vol. 24, , pp. 173 – 186.

Reid, H. F., 1910. The Mechanics of the Earthquake, The California Earthquake of April 18, 1906. Report of the State Investigation Commission, vol. 2. *Carnegie Institution of Washington, Washington, D. C.*.

Rempel, A. W., 2006. The effects of flash-weakening and damage on the evolution of fault strength and temperature. *AGU Monograph on Radiated Energy and the Physics of Earthquake Faulting*, 263-270.

Rempel, A. W., and J. R. Rice, 2006. Thermal pressurization and onset of melting in fault zones. *J. Geophys. Res.* 111, doi:10.1029/2006JB004314.

Rice, J. R., 1999. Flash heating at asperity contacts and rate-dependent friction. *Eos Trans. AGU* 80(46), Fall Meet. Suppl., Abstract F681l.

Rice, J. R., 2006. Heating and weakening of faults during earthquake slip. *J. Geophys. Res.*, doi: 10.1029/2005JB004006.

Rice, J. R. and M. P. Cleary, 1976. Some basic stress-diffusion solutions for fluid saturated elastic porous media with compressible constituents. In *Rev. Geophys. Space Phys.* 14, 227-241.

Rice, J. R. and M. Cocco, 2006. Seismic fault rheology and earthquake dynamics. In *The Dynamics of Fault Zones, 95th Dahlem Workshop*. In press.

Schmidt, E. H. W. and F. Mayinger, 1963. Viscosity of Water and Steam at High Pressures and Temperatures Up to 800 Atmospheres and 700°C. Edited by Warren Ibele. *Modern Developments in Heat Transfer*, p.265.

Scholz, C. H., 1998. Earthquakes and friction laws. *Nature* 391, 37-42.

Scholz, C. H., 2002. The Mechanics of Earthquakes and Faulting. Second edition. *Cambridge University Press.*

Segall, P. and J. R. Rice, 1995. Dilatancy, compaction, and slip instability of a fluid–infiltrated fault. *J. Geophys. Res.* 100, 22155–22171.

Segall, P. and J. R. Rice, 2006. Does shear heating of pore fluid contribute to earthquake nucleation? *J. Geophys. Res.* 111, B09316.

Sibson, R. H., 1973. Interaction between temperature and pore-fluid pressure during earthquake faulting: A mechanism for partial or total stress relief. *Nature* 243, 66–68.

Sibson, R. H., 2003. Thickness of the seismic slip zone. *Bull. Seismol. Soc. Am.* 93, 1169–1178.

Roig Silva, C., D. L. Goldsby, G. di Toro, and T. E. Tullis, 2004.The Role of Silica Content in Dynamic Fault Weakening Due to Gel Lubrication. Abstract. *American Geophysical Union, Fall Meeting 2004.*

Skempton, A. W., 1954. The pore-pressure coefficients A and B. *Giotechnique* 4, No. 4, 143-147.

Sone, H. and T. Shimamoto, 2008. High velocity friction experiments suggesting dynamically weak, but statically strong faults. *Geophysical Research Abstracts*, vol. 10.

Sulem, J., I. Vardoulakis, H. Ouffroukh, M. Boulon, and J. Hans, 2004. Experimental characterization of the thermo-poro-mechanical properties of the Aegion Fault gouge. *C. R. Geosci.* 336(4–5), 455–466.

Sulem, J., I. Vardoulakis, H. Ouffroukh, and V. Perdikatsis, 2005. Thermoporo-mechanical properties of the Aegion fault clayey gouge — application to the analysis of shear heating and fluid pressurization. *Soils Found.* 45(2), 97–108.

Suzuki, T. and T. Yamashita, 2006. Non-linear thermo-poroelastic effects on dynamic earthquake ruptures. *J. Geophys. Res.* 111(B3).

Takahashi, T., K. Tanifuji, C. Stafford and, T. Hashida, 2003. Permeability Enhancement by Microfracturing in Granite under Supercritical Water Conditions. *JSME International Journal Series A* 46, 24-29.

Tembe, S., D. A. Lockner, J. G. Solum, C. A. Morrow, T. Wong, and D. E. Moore, 2006. Frictional strength of cuttings and core from SAFOD drillhole phases 1 and 2. *Geophys. Res. Lett.* 33.

Terzaghi, K., 1923. Die berechnung der durchlassigkeitzifer des tones aus dem verlauf der hydrodynamischen spannungserscheinungen. *Akademie der Wissenschaften, Mathematish-naturwissenschaftliche, Klasse, Vienna*, Part IIa, vol. 132, 125-138.

Tsutsumi, A. and T. Shimamoto, 1997. High velocity frictional properties of gabbro. *Geophys. Res. Lett.* 24, 699–702.

Turcotte, D. L. and G. Schubert, 2002. Geodynamics. Second edition. *Cambridge University Press.*

Ueda, K., T. Komai, I. Yu, and H. Nakayama, 2002. Molecular Dynamics Study on the Density Fluctuation of Supercritical Water. *Journal of Computer Chemistry*, Vol. 1, No. 3, 83-88.

Vafai, K., 2005. Handbook of Porous Media. Second edition. *CRC Press.*

Viesca, R. C., E. L. Templeton, and J. R. Rice, 2008. Off-fault plasticity and earthquake rupture dynamics: 2. Effects of fluid saturation. *J. Geophys. Res.* 113, B09307.

Wang, H. F., 1997. Effects of deviatoric stress on undrained pore pressure response to fault slip. *J. Geophys. Res.* 102(B8), 17943–17950.

Wang, H. F., 2000. Theory of Linear Poroelasticity (with Applications to Geomechanics and Hydrogeology). *Princeton University Press.*

Wang, L., 2000. Flows through Porous Media: A Theoretical Development at Macroscale. *Transport in Porous Media* 39, 1–24.

Weertman, J. J., 1963. Dislocations moving uniformly on the interface between isotropic media of different elastic properties. *J. Mech. Phys. Solids.* 11, 197-204.

Whitaker, S., 1986. Flow in Porous Media I: A Theoretical Derivation of Darcy's Law. *Transport in Porous Media* 1, 3-25.

Wibberley, C. A. J., 2002. Hydraulic diffusivity of fault gouge zones and implications for thermal pressurization during seismic slip. *Earth Planets Space* 54, 1153–1171.

Wibberley, C. A. J. and T. Shimamoto, 2003. Internal structure and permeability of major strike-slip fault zones: The Median Tectonic Line in mid prefecture, southwest Japan. *J. Struct. Geol.* 25, 59-78.

Wibberley, C. A. J. and T. Shimamoto, 2005. Earthquake slip weakening and asperities explained by thermal pressurization. *Nature* 436, 689–692.

Zimmerman, R., W., S. Kumar, and G. S. Bodvarsson, 1991. Lubrication theory analysis of the permeability of rough-walled fractures. *Int. J. Rock Mech.* 28, 325-331.